Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?*

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Abstract

Uncertainty about the future rises in recessions. But is uncertainty a source of business cycle fluctuations or an endogenous response to them, and does the type of uncertainty matter? We find that sharply higher uncertainty about real economic activity in recessions is fully an endogenous response to other shocks that cause business cycle fluctuations, while uncertainty about financial markets is a likely source of the fluctuations. Financial market uncertainty has quantitatively large negative consequences for several measures of real activity including employment, production, and a broad real activity index. Such are the main conclusions drawn from estimation of three-variable structural vector autoregressions (SVAR). To establish dynamic causal effects, we exploit information from external variables and the timing of extraordinary economic events to identify the SVAR under credible interpretations of the structural shocks.
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1 Introduction

A large literature in macroeconomics investigates the relationship between uncertainty and business cycle fluctuations. Interest in this topic has been spurred by a growing body of evidence that uncertainty rises sharply in recessions. This evidence is robust to the use of specific proxy variables such as stock market volatility and forecast dispersion as in Bloom (2009), or a broad-based measure of macroeconomic uncertainty, as in Jurado, Ludvigson, and Ng (2015) (JLN hereafter). But while this evidence substantiates a role for uncertainty in deep recessions, the question of whether uncertainty is an exogenous source of business cycle fluctuations or an endogenous response to economic fundamentals is not fully understood. Existing results are based on convenient but restrictive identifying assumptions and have no explicit role for financial markets, even though the uncertainty measures are strongly correlated with financial market variables. This paper considers a novel identification strategy to disentangle the causes and consequences of real and financial uncertainty.

The question of causality and the identification of exogenous variation in uncertainty is a long-standing challenge of the uncertainty literature. The challenge arises in part because there is no theoretical consensus on whether the uncertainty that accompanies deep recessions is primarily a cause or effect (or both) of declines in economic activity. Theories in which uncertainty is defined as the time varying volatility of a fundamental shock cannot address this question because, by design, there is no feedback response of uncertainty to other shocks if the volatility process is specified to evolve exogenously. And, obviously, models in which there is no exogenous variation in uncertainty cannot be used to analyze the direct effects of uncertainty shocks. It is therefore not surprising that many theories for which uncertainty plays a role in recessions reach contradictory conclusions on this question, as we survey below.

A separate challenge of the uncertainty literature pertains to the origins of uncertainty. Classic theories assert that uncertainty originates from economic fundamentals such as productivity, and that such real economic uncertainty, when interacted with market frictions, discourages real activity. But some researchers have argued that uncertainty dampens the economy through its influence on financial markets (e.g., Gilchrist, Sim, and Zakrajsek (2010)). Moreover, as surveyed by Ng and Wright (2013), all the post-1982 recessions have origins in financial markets, and these recessions have markedly different features from recessions where financial markets play a passive role. From this perspective, if financial shocks are subject to time-varying volatility, financial market uncertainty–as distinct from real economic uncertainty–could be a key player in recessions, both as a cause and as a propagating mechanism. The Great Recession of 2008, characterized by sharp swings in financial markets, hints at such a linkage. Yet so far the literature has not disentangled the contributions of real versus financial uncertainty to business cycle fluctuations.

Econometric analyses aimed at understanding the role of uncertainty for business cycle fluctuations face their own challenges, especially when the body of theoretical work does not provide precise identifying restrictions for empirical work. Attempts to identify the "effects" of uncertainty shocks in existing empirical work are primarily based on recursive schemes within the framework of vector-autoregressions (VAR).[1] But studies differ according to whether uncertainty is ordered ahead of or after real activity variables in the VAR. While a recursive structure is a reasonable starting point, any presumed ordering of the variables is hard to defend on theoretical grounds given the range of models in the literature. Contemporaneous changes in uncertainty can arise both as a cause of business cycle fluctuations and as a response to other shocks. Recursive structures explicitly rule out this possibility since they presume that some variables respond only with a lag to others.

It is with these challenges in mind that we return to the questions posed above: is uncertainty primarily a source of business cycle fluctuations or a consequence of them? And what is the relation of real versus financial uncertainty to business cycle fluctuations? The objective of this paper is to address these questions econometrically using a small-scale structural vector autoregression (SVAR). To confront the challenges just discussed, we take a two-pronged approach. First, our empirical analysis explicitly distinguishes *macro* uncertainty from *financial* uncertainty. The baseline SVAR we study describes the dynamic relationship between three variables: an index of macro uncertainty, U_{Mt}, a measure of real economic activity, Y_t (e.g., production, employment), and a new financial uncertainty index introduced here, U_{Ft}. Second, rather than relying on ordering assumptions for identification, we use a different identification scheme that is less restrictive, both because it allows for simultaneous feedback between uncertainty and real activity, and because it can be used to test whether a lower recursive structure is supported by the data.

Our identification scheme is based in part on the use of external variables that may be related to all three shocks in the SVAR but are presumed on economic grounds to have a non-zero correlation with the structural uncertainty shocks. The approach takes a variable S_t that is not in the SVAR system and solves jointly for two components of S_t, a Z_{1t} that is correlated with macro and financial uncertainty shocks but contemporaneously uncorrelated with real activity shocks, and a Z_{2t} that is correlated with financial uncertainty shocks but contemporaneously uncorrelated with both real activity and macro uncertainty shocks. We refer to these constructed components as "synthetic external variables." They are denoted $Z_{1t}(\boldsymbol{\beta})$ and $Z_{2t}(\boldsymbol{\beta})$ hereafter to emphasize that they are constructed and, as a consequence, depend on a parameter vector $\boldsymbol{\beta}$.

In the present context, the key is to find observables external to our SVAR that are driven

[1]See Bachmann, Elstner, and Sims (2013), Bloom (2009), Bloom (2014), Bekaert, Hoerova, and Duca (2013), Gilchrist, Sim, and Zakrajsek (2010), and JLN.

by a multitude of innovations, including the uncertainty shocks we are interested in. We argue below that both theory and evidence suggest that aggregate stock market returns are natural candidates for such S_t variables. Our maintained economic hypothesis is that stock market returns are correlated with both types of uncertainty shocks and therefore have a Z_{1t} and Z_{2t} component, as described above.

Identification is achieved by combining estimates of $Z_{1t}(\boldsymbol{\beta})$ and $Z_{2t}(\boldsymbol{\beta})$ with restrictions based on economic reasoning. The first economic restriction regards the maintained assumption that uncertainty shocks should be correlated with aggregate stock market returns. Rather than taking a stand on a particular magnitude, bounds are set for the minimum absolute values of these correlations. This amounts to putting restrictions on the correlations of uncertainty shocks with $Z_{1t}(\boldsymbol{\beta})$ and $Z_{2t}(\boldsymbol{\beta})$. We refer to these restrictions as *correlation constraints*. Another economic restriction is that the identified shocks must be consistent with economic reasoning in a small number of extraordinary events, such as the 1987 stock market crash and the financial crisis/Great Recession of 2007-09. We refer to these as "event timing constraints," or simply *event constraints*. We identify sets of solutions that satisfy both correlation and event constraints. Naturally, the sets may be larger or smaller depending on the constraints. We find that, with relatively unrestrictive constraints, the set of solutions to the SVAR identification problem is substantially narrowed to reveal a well defined pattern of dynamic causal effects.

Our use of external variables S_t bears some analogies to the external instrumental variable (IV) approach in the SVAR literature but is distinct from it. The difference is that the external variables S_t are not themselves presumed to be valid exogenous instruments. Instead, components of observable external variables S_t are presumed on economic grounds to exhibit a non-zero correlation with the structural uncertainty shocks and have the exogeneity properties of Z_{1t} and Z_{2t}. We estimate these S_t components using an iterative approach that we refer to as *iterative projection external variable* (IPEV). We show that this approach can be fit into the classic simultaneous equations framework and interpreted as the output of a restricted system estimation for a larger VAR that includes both $\mathbf{X}_t$ and S_t.

Our use of event constraints also differs from the extant SVAR literature, in which shocks are constructed primarily as a means to other ends, such as the computation of impulse response functions and variance decompositions. By contrast, we study the estimated shocks in detail and use their behavior at specific points in the historical sample as an identification tool.

The empirical exercise additionally requires that appropriate measures of macro and financial uncertainty be available. To this end, we exploit a data rich environment, working with 134 macro monthly time series and 147 financial variables. The construction of macro uncertainty follows JLN. The same approach is used to construct a broad-based measure of financial uncertainty that has never been used in the literature. Macro uncertainty is itself an aggregate of uncertainties in variables from three categories: real activity, price, and financial. To better understand the contributions of each of these categories, we also replace U_{Mt} in the VAR with an uncertainty measure based on the real activity sub-component. Uncertainty about real activity is of special interest because classic uncertainty theories postulate that uncertainty shocks have their origins in economic fundamentals and hence should show up as uncertainty about real economic activity.

Before summarizing our main results, it should be made clear that the structural shocks we identify do not necessarily correspond to shocks of any particular model, as this is not our goal. Our real activity shocks could be a combination of technology, monetary policy, preferences, government expenditures, and our macro uncertainty shocks can originate from economic policies and/or technology. Given our view that the questions raised above are ultimately empirical ones, our goal is to use a model-free approach that can identify the dynamic causal effects of macro and financial uncertainty shocks when commonly used ordering or timing assumptions are difficult to defend. The objective is to establish a set of stylized facts on the dynamic causal relationships among these variables, against which a wide range of individual models could be evaluated.

Our main results may be stated as follows. First, positive shocks to financial uncertainty are found to cause a sharp decline in real activity that persists for many months, lending support to the hypothesis that heightened uncertainty is an exogenous impulse that causes recessions. These effects are especially large for several measures of real activity, notably production, employment and a broad real activity index. The finding that heightened uncertainty has negative consequences for real activity is qualitatively similar to that of preexisting empirical work that uses recursive identification schemes (e.g., Bloom (2009), JLN), but differs in that we trace the source of this result specifically to broad-based financial market uncertainty rather than to various uncertainty proxies or broad-based macro uncertainty. We also show that the converse is not supported by our evidence: exogenous shocks to real activity have no clear effect on financial uncertainty given the set of SVAR parameters we identify.

Second, the identification scheme used here reveals something new that is not possible to uncover under recursive schemes: macro and financial uncertainty have a very different dynamic relationship with real activity. Specifically, unlike financial uncertainty, sharply higher macro and real activity uncertainty in recessions is found to be an endogenous response to business cycle fluctuations. That is, negative economic activity shocks are found to cause increases in both macro uncertainty and in the sub-index that measures uncertainty about real economic activity, but there is no evidence that independent shocks to macro or real activity uncertainty cause lower economic activity. Indeed the opposite is often true: exogenous shocks to both macro and real uncertainty are found to *increase* real activity, consistent with "growth options" theories discussed below.

Third, our results are distinct from those obtained using recursive identification. Under any recursive ordering of the variables in our VAR, exogenous shocks that increase macro or real uncertainty appear to reduce real activity, in a manner that is qualitatively similar to financial uncertainty shocks. This result does not hold in the less restrictive SVAR studied here and appears to be an artifact of invalid timing assumptions under recursive identification. Further investigation reveals that the SVAR we study reflects a non-zero contemporaneous correlation between U_{Ft} and Y_t, as well as between U_{Mt} and Y_t, which is inconsistent with any recursive ordering. Tests of the validity of a recursive structure are easily rejected by the data.

The rest of this paper is organized as follows. Section 2 reviews related literature and provides motivation for our maintained economic hypothesis that stock market returns have components that are correlated with macro and financial uncertainty shocks but contemporaneously uncorrelated with real activity, and also correlated with financial uncertainty but contemporaneously uncorrelated with both real activity and macro uncertainty. Section 3 details the econometric framework and identification employed in our study, describes how the synthetic external variables are constructed, and discusses the data and empirical implementation. In this section we also show how, with some additional restrictions, our approach can interpreted as the output of a system estimation for a larger VAR that includes both $\mathbf{X}_t$ and S_t. Section 4 presents empirical results using broad based macro uncertainty U_{Mt}, while Section 5 reports results for systems that isolate the sub-component of U_{Mt} corresponding to real activity variables. Section 6 reports results pertaining to robustness and additional cases. In this section we consider an estimation where we take two observed external variables S_{1t} and S_{2t} and presume they are valid external instruments Z_{1t} and Z_{2t}. This is compared to the case where the same variables are presumed not to be valid instruments and IPEV is used to construct synthetic instruments from S_{1t} and S_{2t}. Section 7 summarizes and concludes. A large number of additional results and Monte Carlo simulations are presented in Ludvigson, Ma, and Ng (2016).

2 Related Literature

A large literature addresses the question of uncertainty and its relation to economic activity.[2] Besides the evidence cited above for the U.S., Nakamura, Sergeyev, and Steinsson (2012) estimate growth rate and volatility shocks for 16 developed countries and find that they are substantially negatively correlated. Theories for which uncertainty plays a key role differ widely on the question of whether this correlation implies that uncertainty is primarily a cause or a consequence of declines in economic activity. In most cases, it is modeled either as a cause or an consequence, but not both.

The first strand of the literature proposes uncertainty as a cause of lower economic growth.

[2]This literature has become voluminous. See Bloom (2014) for a recent review of the literature.

This includes models of the real options effects of uncertainty (Bernanke (1983), McDonald and Siegel (1986)), models in which uncertainty influences financing constraints (Gilchrist, Sim, and Zakrajsek (2010), Arellano, Bai, and Kehoe (2011)), or precautionary saving (Basu and Bundick (2012), Leduc and Liu (2012), Fernández-Villaverde, Pablo Guerrón-Quintana, and Uribe (2011)). These theories almost always presume that uncertainty is an exogenous shock to some economic fundamental. Some theories presume that higher uncertainty originates directly in the process governing technological innovation, which subsequently causes a decline in real activity (e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012)).

A second strand of the literature postulates that higher uncertainty arises solely as a *response* to lower economic growth, emphasizing a variety of mechanisms. Some of these theories suggest that bad times incentivize risky behavior (Bachmann and Moscarini (2011), Fostel and Geanakoplos (2012)), or reduce information and with it the forecastability of future outcomes (Van Nieuwerburgh and Veldkamp (2006) Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014)), or provoke new and unfamiliar economic policies whose effects are highly uncertain (Pástor and Veronesi (2013)), or create a greater misallocation of capital across sectors (Ai, Li, and Yang (2015)), or generate endogenous countercyclical uncertainty in consumption growth because investment is costly to reverse (Gomes and Schmid (2016)).

And yet a third literature has raised the possibility that some forms of uncertainty can actually *increase* economic activity. "Growth options" theories of uncertainty postulate that a mean-preserving spread in risk generated from an unbounded upside coupled with a limited downside can cause firms to invest and hire, since the increase in mean-preserving risk increases expected profits. Such theories were often used to explain the dot-com boom. Examples include Bar-Ilan and Strange (1996), Pastor and Veronesi (2006), Kraft, Schwartz, and Weiss (2013), Segal, Shaliastovich, and Yaron (2015).

This brief review reveals a rich literature with a wide range of predictions about the relationship between uncertainty and real economic activity. Yet the absence of a theoretical consensus on this matter, along with the sheer number of theories and limited body of evidence on the structural elements of specific models, underscores the extent to which the question of cause and effect is fundamentally an empirical matter that must be settled in an econometric framework with as little specific theoretical structure as possible, so that the various theoretical possibilities can be nested in empirical tests. Commonly used recursive identification schemes cannot achieve this objective, since by construction they rule out the possibility that uncertainty and real activity could influence one another within the period. Our econometric model nests any recursive identification scheme, so we can test whether such timing assumptions are plausible. We find they are rejected by the data.

Our maintained hypothesis that stock market returns should be correlated with uncertainty shocks builds on work in asset pricing emphasizing the idea that stock market variation is the result of several distinct (and orthogonal) sources of stochastic variation. For example, one quantitatively important component is attributable to *a*cyclical risk premia variation, and more generally appears to be uncorrelated with most measures of real activity.[3] This component is valuable for our objective because it is exogenous to real activity, but may still be relevant for both macro and financial uncertainty, as in our synthetic Z_{1t}. Yet another component could be attributable to fluctuations in factors like corporate leverage, or in the risk aversion or "sentiment" of market participants that may be correlated with the volatility of the stock market. In equilibrium asset pricing models, if leverage increases, volatility of the corporate sector's equity return increases. Thus changes in factors like leverage (and possibly changes in risk aversion or sentiment) should be correlated with financial uncertainty, but may have little to do with uncertainty about economic fundamentals. This component is valuable for our objective because it is plausibly uncorrelated with both real activity and uncertainty about economic fundamentals, but may still be relevant for financial market uncertainty, as in our synthetic Z_{2t}. Consistent with the existence of this type of component, JLN document that there are many spikes in stock market uncertainty that do not coincide with an important movement in either real activity or macro uncertainty. These findings motivate our maintained hypothesis that measures of equity market returns are promising non-uncertainty variables comprised of several distinct sources of stochastic variation, two of which have the statistical characteristics of a Z_{1t} and Z_{2t}.

Our IPEV approach is related to a recent line of econometric research in SVARs that uses information contained in external instruments to identify structural dynamic causal effects.[4] Of these, Stock and Watson (2012) study uncertainty shocks, using a measure of stock market volatility and/or a news media measure of policy uncertainty from Baker, Bloom, and Davis (2013), as separate external instruments for identifying the effects of uncertainty shocks in a SVAR. Our study differs in some fundamental ways. First, our approach relies on a set of economic assumptions that is distinct from that of standard IV approach, hence the moment conditions used to identify the model parameters and shocks are not the same. The identification strategy in Stock and Watson (2012) for uncertainty shocks presumes that the external variables themselves (i.e., stock market volatility, policy uncertainty) are valid instruments, correlated with the uncertainty shock of interest but not with the other shocks. By contrast, our approach explicitly views both the stock market and our uncertainty measures as partly endogenous, forcing us to confront the identification quandary. Our identification assumption is instead

[3]For empirical evidence, see Lettau and Ludvigson (2013), Greenwald, Lettau, and Ludvigson (2014), Kozak and Santosh (2014), and Muir (2014). Theoretical examples include Bansal and Yaron (2004); Wachter (2013); Gourio (2012); Brunnermeier and Sannikov (2012); Bianchi, Ilut, and Schneider (2014); Gabaix and Maggiori (2013); He and Krishnamurthy (2013). These papers have some form of uncertainty shock to economic fundamentals that drives risk premia.

[4]See for example Hamilton (2003), Kilian (2008), Mertens and Ravn (2013); Stock and Watson (2008), Stock and Watson (2012), and Olea, Stock, and Watson (2015).

that aggregate stock market returns contain components that are correlated with the structural uncertainty shocks and we consider solutions to the SVAR identification problem that rely on lower bounds for the absolute values of these correlations. Second, Stock and Watson (2012) focus exclusively on identifying the effects of uncertainty shocks and do not attempt to simultaneously identify the converse, namely the effects of real activity shocks on uncertainty.

Berger, Dew-Becker, and Giglio (2016) take a different approach. Using options data they find that bad times are associated with higher realized volatility but not higher expected volatility, a result that they interpret as consistent with the hypothesis that higher uncertainty is a consequence of negative economic shocks rather than a cause. This interpretation is not intended to provide an explicit identification of uncertainty shocks, however.

The study arguably closest in spirit to our identification approach is Baker and Bloom (2013), who use disaster-like events as instruments for stock market volatility with the aim of isolating exogenous variation in uncertainty. This has some similarities with our approach, in that it implicitly assumes that certain components of stock market fluctuations (those associated with "disasters") are exogenous. In contrast to our approach, disasters chosen subjectively are presumed to be valid instruments for uncertainty, whereas we instead use external stock return data and unusual events to constrain a set of estimable moment restrictions. It is of interest that we arrive at complementary conclusions, despite the differing methodologies for identifying exogenous variation.

3 Econometric Framework

This section outlines our econometric approach. Subsection 1 explains the identification strategy. Subsections 2 and 3 explain the construction of external instruments in the IPEV procedure and the uncertainty measures. Subsection 4 shows how our approach can be fit into the classic simultaneous equations framework and interpreted as the output of a restricted system estimation for a larger VAR that includes both $\mathbf{X}_t$ and S_t.

3.1 The SVAR and Identification

Let $\mathbf{X}_t$ denote a $K \times 1$ vector time series. We suppose that $\mathbf{X}_t$ has a reduced-form vector autoregressive and an infinite moving average representation given respectively by:

$$\begin{aligned} \mathbf{X}_t &= \mathbf{k} + \mathbf{A}_1\mathbf{X}_{t-1} + \mathbf{A}_2\mathbf{X}_{t-2} + \cdots + \mathbf{A}_p\mathbf{X}_{t-p} + \boldsymbol{\eta}_t. \qquad (1) \\ \mathbf{X}_t &= \boldsymbol{\mu} + \boldsymbol{\Psi}(L)\,\boldsymbol{\eta}_t \qquad (2) \\ \boldsymbol{\eta}_t &\sim (0, \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \mathbb{E}(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') \end{aligned}$$

where $\boldsymbol{\Psi}(L) = \mathbf{I}_n + \boldsymbol{\Psi}_1 L + \boldsymbol{\Psi}_2 L^2 + \ldots$ is a polynomial in the lag operator L of infinite order, $\boldsymbol{\Psi}_s$ is the $(n \times n)$ matrix of coefficients for the sth lag of $\boldsymbol{\Psi}(L)$. The reduced form innovations

$\boldsymbol{\eta}_t$ are related to the structural shocks $\mathbf{e}_t$ by an invertible $K \times K$ matrix $\mathbf{H}$:

$$\boldsymbol{\eta}_t = \mathbf{H\Sigma e}_t \equiv \mathbf{Be}_t \tag{3}$$

$$\mathbf{e}_t \sim (0, \mathbf{I_K}), \qquad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{11} & 0 & \cdot & 0 \\ 0 & \sigma_{22} & 0 & 0 \\ 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \sigma_{KK} \end{pmatrix}, \quad \sigma_{jj} \geq 0 \; \forall j. \tag{4}$$

where $\mathbf{B} \equiv \mathbf{H\Sigma}$. The structural shocks $\mathbf{e}_t$ are mean zero with unit variance, and are serially and mutually uncorrelated. A normalization is required to pin down the sign and scale of the shocks. We adopt the unit effect normalization

$$\operatorname{diag}(\mathbf{H}) = 1. \tag{5}$$

The objective of the exercise is to study the dynamic effects and the relative importance of each structural shock j. These are summarized by the impulse response function (IRF) $\frac{\partial \mathbf{X}_{t+s}}{\partial e_{jt}} = \boldsymbol{\Psi}_s \mathbf{b}^j$ (where $\mathbf{b}^j$ is the jth column of $\mathbf{B}$) and the fraction of s-step ahead forecast error variance of $\mathbf{X}_t$ that is attributable to each structural shock. The SVAR identification problem concerns identifying the elements of $\mathbf{H}$ and $\boldsymbol{\Sigma}$, from which the structural IRFs and variance decompositions are computed.

To study the impulse and propagating mechanism of uncertainty shocks while explicitly distinguishing between macro and financial market uncertainty, we consider a system with $K = 3$ variables. Our baseline SVAR is based on $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$, where U_{Mt} denotes macro uncertainty, Y_t denotes a measure of real activity, and U_{Ft} denotes financial uncertainty. The corresponding reduced form shocks $\boldsymbol{\eta}_t = (\eta_{Mt}, \eta_{Yt}, \eta_{Ft})'$ are related to the three structural form shocks $\mathbf{e}_t = (e_{Mt}, e_{Yt}, e_{Ft})'$ for macro uncertainty, real activity, and financial uncertainty, as follows:

$$\begin{aligned} \eta_{Mt} &= B_{MM} e_{Mt} + B_{MY} e_{Yt} + B_{MF} e_{Ft} \\ \eta_{Yt} &= B_{YM} e_{Mt} + B_{YY} e_{Yt} + B_{YF} e_{Ft} \\ \eta_{Ft} &= B_{FM} e_{Mt} + B_{FY} e_{Yt} + B_{FF} e_{Ft}, \end{aligned}$$

where B_{ij} is the element of $\mathbf{B}$ that gives the contemporaneous effect of the jth structural shock on the ith variable. The covariance structure of $\boldsymbol{\eta}_t$ provides $K(K+1)/2 = 6$ equations in $\mathbf{B}$:

$$\operatorname{vech}(\boldsymbol{\Omega}) = \operatorname{vech}(\mathbf{BB}') \tag{6}$$

where $\operatorname{vech}(\boldsymbol{\Omega})$ stacks the unique elements of the symmetric matrix $\boldsymbol{\Omega}$. There are nine unknown elements in $\mathbf{B}$.

To motivate our procedure, it is helpful to begin by considering the external IV approach where valid instruments are observed. To do so, suppose for the moment that we have measures of Y_t, U_{Mt}, U_{Ft}, and two external instruments, Z_{1t} and Z_{2t} satisfying the following:

Assumption A: Let $\mathbf{Z}_t = (Z_{1t}, Z_{2t})'$ be two instrumental variables such that

$$\begin{array}{llll} (A.i) & \mathbb{E}[Z_{1t}e_{Mt}] \neq 0, & \mathbb{E}[Z_{1t}e_{Yt}] = 0, & E[Z_{1t}e_{Ft}] \neq 0 \\ (A.ii) & \mathbb{E}[Z_{2t}e_{Mt}] = 0, & \mathbb{E}[Z_{2t}e_{Yt}] = 0, & \mathbb{E}[Z_{2t}e_{Ft}] \neq 0. \end{array}$$

Assumption A are conditions for instrument exogeneity and relevance. Z_{1t} is an instrument that is correlated with both macro and financial uncertainty, but contemporaneously uncorrelated (exogenous) with respect to real activity. Z_{2t} is an instrument that is correlated with financial uncertainty, but contemporaneously uncorrelated (exogenous) with respect to macro uncertainty and real activity.

Let $\mathbf{m}_{1t} = (\text{vech}(\boldsymbol{\eta}_t\boldsymbol{\eta}_t'), \text{vec}\,(\mathbf{Z}_t \otimes \boldsymbol{\eta}_t))'$ and $\boldsymbol{\beta} = \text{vec}(\mathbf{B})$. At the true value of $\boldsymbol{\beta}$, denoted $\boldsymbol{\beta}^0$, the model satisfies

$$0 = \mathbb{E}[\mathbf{g}_1(\mathbf{m}_{1t}; \boldsymbol{\beta}^0)], \tag{7}$$

written out in full as follows:

$$\begin{aligned}
0 &= var(\eta_M) - B_{MM}^2 + B_{MY}^2 + B_{MF}^2 \\
0 &= var(\eta_Y) - B_{YM}^2 + B_{YY}^2 + B_{YF}^2 \\
0 &= var\,(\eta_F) - B_{FM}^2 + B_{FY}^2 + B_{FF}^2 \\
0 &= cov(\eta_M, \eta_Y) - B_{MM}B_{YM} + B_{MY}B_{YY} + B_{MF}B_{YF} \\
0 &= cov(\eta_Y, \eta_F) - B_{YM}B_{FM} + B_{YY}B_{FY} + B_{FF}B_{YF} \\
0 &= cov\left(\eta_{M,}\eta_F\right) - B_{MM}B_{FM} + B_{MY}B_{FY} + B_{MF}B_{FF} \\
0 &= B_{MF}\mathbb{E}[Z_{2t}\eta_Y] - B_{YF}\mathbb{E}[Z_{2t}\eta_{Mt}] \\
0 &= B_{FF}\mathbb{E}[Z_{2t}\eta_{Yt}] - B_{YF}\mathbb{E}[Z_{2t}\eta_{Ft}] \\
0 &= (B_{MM}B_{FF} - B_{MF}B_{FM})\mathbb{E}[Z_{1t}\eta_{Yt}] - (B_{YF}B_{FM} - B_{YM}B_{FF})\mathbb{E}[Z_{1t}\eta_{Mt}] \\
&\quad -(B_{MM}B_{YF} - B_{MF}B_{YM})\mathbb{E}[Z_{1t}\eta_{Ft}].
\end{aligned}$$

The model has nine equations in nine unknowns. The first six are from the covariance structure. The next two equations are due to the three moments implied by Assumption $(A.ii)$. The final equation is due to the three moments implied by Assumption $(A.i)$.

Proposition 1 *Under Assumption A with $det(B) > 0$, the normalization (5), and the restriction (4), $\boldsymbol{\beta}$ is identified.*

In essence, identification in this analysis is achieved by (i) using movements in U_{Mt} and U_{Ft} that are correlated with Z_{1t} to identify the effects of uncertainty shocks and disentangle them from shocks to real activity, (ii) using movements in U_{Ft} that are correlated with Z_{2t} to identify the effects of U_{Ft} shocks and disentangle them from macro uncertainty shocks, and (iii) using movements in Y_t that are uncorrelated with both Z_{1t} and Z_{2t} to identify the effects of real activity shocks and disentangle them from uncertainty shocks.[5] The Appendix gives a closed-form solution for $\mathbf{B}$, and shows that the covariance between the instruments and the structural shocks can be expressed as

$$\begin{aligned}
\mathbb{E}[Z_{2t}e_{Ft}]^2 &= \mathbb{E}[\boldsymbol{\eta}_t Z_{2t}]'\boldsymbol{\Omega}^{-1}\mathbb{E}[\boldsymbol{\eta}_t Z_{2t}] \\
\mathbb{E}[Z_{1t}e_{Mt}]^2 &= \left(\mathbb{E}[\boldsymbol{\eta}_t Z_{1t}] - \frac{\mathbb{E}[\boldsymbol{\eta}_t Z_{2t}]}{\mathbb{E}[Z_{2t}e_{Ft}]}\mathbb{E}[Z_{2t}e_{Ft}]\right)'\boldsymbol{\Omega}^{-1}\left(\mathbb{E}[\boldsymbol{\eta}_t Z_{1t}] - \frac{\mathbb{E}[\boldsymbol{\eta}_t Z_{2t}]}{\mathbb{E}[Z_{2t}e_{Ft}]}\mathbb{E}[Z_{2t}e_{Ft}]\right) \\
\mathbb{E}[Z_{2t}e_{Ft}]\mathbb{E}[Z_{1t}e_{Ft}] &= \mathbb{E}[\boldsymbol{\eta}_t Z_{2t}]'\boldsymbol{\Omega}^{-1}\mathbb{E}[\boldsymbol{\eta}_t Z_{1t}].
\end{aligned}$$

Since we take the stand in this application that our uncertainty measures are potentially endogenous, it is then natural to ask why we do not simply find observable instruments. One answer is that credible valid instruments for uncertainty that are truly exogenous may be difficult or impossible to find and defend. Indeed, existing uncertainty proxies are likely to be among those variables that fall into this category. Beyond this, JLN find that many popular uncertainty proxies, including options-based volatility indexes such as VIX or VXO, are less defensible measures of uncertainty than those employed here, so it makes little sense to instrument for the latter with the former. Options-based volatility indexes are doubly problematic for our purpose because they are known to contain a large component attributable to changes in the variance risk premium that are unrelated to common notions of uncertainty (e.g., Bollerslev, Tauchen, and Zhou (2009); Carr and Wu (2009)).[6] With these considerations in mind, the next subsection proposes a methodology to construct synthetic proxy variables that satisfy the exogeneity properties of valid instruments and can be combined with additional economic restrictions to identify dynamic causal effects in the SVAR for $\mathbf{X}_t$.

3.2 Construction of Synthetic Proxies

Suppose that the external instruments Z_{1t} and Z_{2t} have no credible observable counterparts. The next step is to develop a methodology to construct synthetic proxies in the spirit of such variables. To motivate our method, recall that two stage least squares uses projections to purge the endogenous variations from a relevant regressor. Our approach is similar except that we purge the endogenous variations from a observed variable that is not of first order relevance to our VAR system. The output of such a projection is a generated or synthetic external "instrument."

In the present context, we make use of observable variables S_t that are driven by the structural shocks $\mathbf{e}_t = (e_{Yt},\ e_{Mt}$ and $e_{Ft})'$, as well as other shocks collected into an e_{St} that are

[5] We verify that the closed-form solution is the the same as the unique numerical solution obtained with (5) and (4) imposed.

[6] This does not preclude the possibility that options based indexes may be valuable in empirical contexts different from ours, such as those that seek to distinguish expected stock market volatility from realized stock market volatility (Berger, Dew-Becker, and Giglio (2016)).

uncorrelated with $\mathbf{e}_t$. A theoretical premise of the paper is that structural uncertainty shocks should be reflected in aggregate equity return variation. Thus our choice of S_t is a measure of stock market returns. As there are many measures of stock returns, we generically denote stock returns by S_t and explicitly distinguish the different measures by S_{1t} and S_{2t} only when the context makes this necessary. Under our maintained assumption, we may represent S_t as

$$S_t = \delta_0 + \delta_Y Y_t + \delta_M U_{Mt} + \delta_F U_{Ft} + \delta_S(L) S_{t-1} + \delta_X(L)'\mathbf{X}_{t-1} + e_{St} \tag{8}$$

where $\mathbf{X}_t = (Y_t, U_{Mt}, U_{Ft})'$. The residual e_{St} could be driven by any number of shocks orthogonal to $\mathbf{e}_t$. One interpretation is risk premium shocks driven by factors orthogonal to uncertainty, such as a pure sentiment shock (one uncorrelated with uncertainty), but the precise interpretation is not important to what follows. It is clear that S_t and $\mathbf{X}_t$ are endogenous variables and least squares estimation of (8) will yield inconsistent estimates. However, we are not interested in these parameters. Our objective in considering stock-market returns is solely to remove from it those variations due to the estimated e_{Yt} or e_{Yt} and e_{Mt} jointly. More precisely, (8) motivates two non-structural representations of S_t:

$$S_{1t} = d_{10} + d_{1Y} e_{Yt} + d_{1S}(L) S_{1t-1} + Z_{1t} \tag{9a}$$

$$S_{2t} = d_{20} + d_{2M} e_{Mt} + d_{2Y} e_{Yt} + d_{2S}(L) S_{2t-1} + Z_{2t}, \tag{9b}$$

where S_{1t} and S_{2t} are not necessarily the same variable. Equation (9a) forms an orthogonal decomposition of S_{1t} into a component that is spanned by e_{Yt} and a component Z_{1t} that is orthogonal to e_{Yt}. Similarly, equation (9b) purges the effect of e_{Yt} and e_{Mt} from S_{2t} to arrive at Z_{2t}. Note that Z_{1t} and Z_{2t} include the effects of $\mathbf{X}_{t-1}$. Moreover, they are forecastable since both U_{Mt} and U_{Ft} can be serially correlated and their lagged values predict future excess stock market returns.

If e_Y and e_M were observed, then solving for the sample analog of (7) would produce estimates of Z_{1t} and Z_{2t} that satisfy Assumption A by construction. Alternatively, if valid instruments $\mathbf{Z}_t$ were observed, Proposition 1 shows that we could identify $\mathbf{B}$, hence $\mathbf{e}_t$. Since both are unobserved, such regressions are infeasible. However, components of observed variables S_t may have the correlation properties of Z_{1t} and Z_{2t} stipulated in Assumption A. Given the theory and evidence discussed above, our maintained hypothesis is that stock market returns are correlated with all three structural shocks $\mathbf{e}_t$, hence they contain a Z_{1t} component that is correlated with both uncertainty shocks but uncorrelated with real activity shocks, and another component Z_{2t} that is correlated with financial uncertainty shocks but uncorrelated with both real activity and macro uncertainty shocks. To the extent that we can identify such components by requiring that they satisfy the same nine equations described in (7), we interpret them as synthetic external variables that proxy for external instruments. We denote these constructed components $\mathbf{Z}(\boldsymbol{\beta})$ to emphasize that they are functions of parameters $\boldsymbol{\beta}$ to be estimated.

But unlike the classic external IV case where $\mathbf{Z}$ is observed, the nine moment restrictions in (7) cannot by themselves identify the SVAR parameters and shocks. Whereas $\mathbf{Z}'\boldsymbol{\eta}$ is fixed when $\mathbf{Z}$ is observed, this is no longer the case when $\mathbf{Z}$ is constructed because $\mathbf{Z}$ itself depends on $\boldsymbol{\beta}$. The problem that this creates is that if $\hat{\mathbf{Z}}_1'\hat{\mathbf{e}}_Y = 0$, $\hat{\mathbf{Z}}_2'\hat{\mathbf{e}}_Y = 0$, and $\hat{\mathbf{Z}}_2'\hat{\mathbf{e}}_M = 0$ for some $\hat{\boldsymbol{\beta}} = \text{vec}(\hat{\mathbf{B}})$, any orthonormal rotation of $\hat{\mathbf{B}}$ to $\tilde{\mathbf{B}} = \hat{\mathbf{B}}\mathbf{Q}'$ and $\hat{\mathbf{e}}$ to $\tilde{\mathbf{e}} = \mathbf{Q}\hat{\mathbf{e}}$ will have $\tilde{\mathbf{Z}}_1'\tilde{\mathbf{e}}_Y = 0$, $\tilde{\mathbf{Z}}_2'\tilde{\mathbf{e}}_Y = 0$, and $\tilde{\mathbf{Z}}_2'\tilde{\mathbf{e}}_M = 0$. This is because the three exogeneity conditions hold by construction; they are imposed to arrive at the nine equations. If we collect all the solutions that satisfy (7) into the set $\hat{\mathcal{B}}$, this set can be infinitely large.

To address this problem, we combine the nine moment restrictions in (7) using the synthetic $Z_{1t}(\boldsymbol{\beta})$ and $Z_{2t}(\boldsymbol{\beta})$ with economic restrictions. The first economic restriction regards the maintained assumption that uncertainty shocks should be correlated aggregate stock market returns. Given a specification for stock returns like (8), this is isomorphic to assuming that the uncertainty shocks must be correlated with the components $Z_{1t}(\boldsymbol{\beta}) = S_{1t} - d_{10} - d_{1Y}e_{Yt} - d_{1S}(L)S_{1t-1}$ and $Z_{2t}(\boldsymbol{\beta}) = S_{2t} - d_{20} - d_{2M}e_{Mt} - d_{2Y}e_{Yt} - d_{2S}(L)S_{2t-1}$.

Our maintained economic hypothesis is that stock market returns must have a non-zero correlation with the structural uncertainty shocks. But how highly correlated? Rather than taking a stand on a particular magnitude, lower bounds are set for the absolute correlations. These bounds act to shrink the unconstrained set because the observed correlations $\mathbf{Z}_1'(\hat{\boldsymbol{\beta}})\hat{e}_M \neq 0$, $\mathbf{Z}_1'(\hat{\boldsymbol{\beta}})\hat{e}_F \neq 0$, $\mathbf{Z}_2'(\hat{\boldsymbol{\beta}})\hat{e}_F \neq 0$ are not invariant to orthonormal rotations. This follows because the $\mathbf{Z}_t$ are constructed from data S_t, which is given. Hence $\hat{\mathbf{B}}$ and $\tilde{\mathbf{B}} = \hat{\mathbf{B}}\mathbf{Q}'$ will imply components $\mathbf{Z}(\boldsymbol{\beta})$ with different correlations.

The second set of economic assumptions requires that the identified shocks in a small number of extraordinary events can be given a defensible interpretation. For this application, we require that the financial uncertainty shocks identified in October 1987 and during the 2007-2009 financial crisis be large and positive, and that the identified output shocks during the Great Recession not take on large positive values. Parameter estimates that suggest favorable financial uncertainty and/or output shocks during particular episodes that are ex-post widely regarded as characterized by either sub-par economic conditions and/or extreme volatility in the stock market are dismissed. Observe that these event timing restrictions also act to shrink the unconstrained set because, while $\tilde{e}_t$ and $\hat{e}_t$ have the same mean and variance, $\tilde{e}_t \neq \hat{e}_t$ at any particular t.

Both sets of economic assumptions are used to dismiss solutions in $\hat{\mathcal{B}}$ to form a winnowed set of solutions $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \overline{k})$, where $\bar{c}, \bar{C}$, and $\overline{k} \equiv (k_1, k_2, k_3)'$ are defined below.

Assumption B: Winnowing Constraints For any $\boldsymbol{\beta} \in \hat{\mathcal{B}}$ that satisfies the nine equations defined in (7) with $\mathbf{Z}(\boldsymbol{\beta})$ constructed as in (9a) and (9b), $\boldsymbol{\beta} \in \overline{\mathcal{B}}(\bar{c}, \bar{C}, \overline{k})$ only if all the following conditions are satisfied:

1 **Correlation constraints**: Let $c_{kj}(\boldsymbol{\beta}) = \text{corr}(Z_{kt}(\boldsymbol{\beta}), e_{jt}(\boldsymbol{\beta}))$ be the sample correlation between $\mathbf{Z}_k(\boldsymbol{\beta})$ and the shock in $\mathbf{e}_t(\boldsymbol{\beta}) = (e_{Mt}, e_{Yt}, e_{Ft})$ with label j.

i $|c_{1M}(\boldsymbol{\beta})| > \bar{c}$, $|c_{1F}(\boldsymbol{\beta})| > \bar{c}$, and $|c_{2F}(\boldsymbol{\beta})| > \bar{c}$.

ii For $c(\boldsymbol{\beta})= \big(c_{1M}(\boldsymbol{\beta}), c_{1F}(\boldsymbol{\beta}), c_{2F}(\boldsymbol{\beta})\big)'$, $\sqrt{c(\boldsymbol{\beta})'c(\boldsymbol{\beta})} > \bar{C}$.

2 **Event constraints**: For $\mathbf{e}_t(\boldsymbol{\beta}) = \mathbf{B}^{-1}\boldsymbol{\eta}_t$,

i $e_{Ft_1}(\boldsymbol{\beta}) > \bar{k}_1$ where t_1 is the period 1987:10 of the stock market crash.

ii There exists a $t_2 \in$ [2007:12, 2009:06] such that $e_{Ft_2}(\boldsymbol{\beta}) > \bar{k}_2$.

iii For all $t_2 \in [2007{:}12, 2009{:}06]$, $e_{Yt_2}(\boldsymbol{\beta}) < \bar{k}_3$

The first type of winnowing constraint pertains to the assumption that stock market returns should be correlated with structural uncertainty shocks and requires that each correlation $c_{1M}(\hat{\boldsymbol{\beta}}), c_{1F}(\hat{\boldsymbol{\beta}}), c_{2F}(\hat{\boldsymbol{\beta}})$ individually exceeds a pre-specified $\bar{c}$, and collectively exceeds $\bar{C}$. The second type of winnowing constraint, based on the timing of unusual events, are used to ensure that the identified shocks at specific episodes have properties that agree with economic reasoning. The t_2 dates are set in accordance with NBER dating of the Great Recession, which coincides with the timing of the financial crisis.

We identify sets of solutions that satisfy both types of winnowing constraints. Naturally, the sets may be larger or smaller depending on how restrictive are the bounds $\bar{c}, \bar{C}, \overline{k}$. If the bounds are unrestrictive, many $\hat{\boldsymbol{\beta}}$ in $\hat{\mathcal{B}}$ will also be in $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \overline{k})$ and little progress is made. If the bounds are too restrictive, no solutions will satisfy the constraints and $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \overline{k})$ will be empty.

It remains to discuss the construction of the unconstrained solution set $\hat{\mathcal{B}}$. To obtain a set of, say, K solutions, we solve K GMM problems with different staring values. Each GMM problem consists of solving for nine unknowns from nine equations, with $\mathbf{Z}$ generated from (9a) and (9b), with an initial guess for the shocks $\mathbf{e}_t$. The guess of $\mathbf{e}_t$ is updated each time $\mathbf{B}$ is updated during each iteration. We have found it more efficient to solve for $\mathbf{B}$ and $\mathbf{Z}$ iteratively, updating $\mathbf{Z}$ by projection only when a solution for $\mathbf{B}$ is obtained. The leads to what we refer to as the *iterative projection external variable* (IPEV) procedure, summarized as follows:

Algorithm IPEV For a given guess of $\boldsymbol{\beta}$ =vec($\mathbf{B}$)and therefore a guess of $\mathbf{e}_t = \mathbf{B}^{-1}\boldsymbol{\eta}_t$, the following steps are repeated until convergence:

i Put the guess of $(\mathbf{e_M}, \mathbf{e_Y})$ in (9a) and (9b) to construct $\mathbf{Z}_1$ and $\mathbf{Z}_2$.

ii Use $\mathbf{Z_1}$ and $\mathbf{Z_2}$ to solve the nine equations given in (7). This gives a new value for $\mathbf{B}$.

iii Construct new shocks $\mathbf{e}$ using the new estimate of $\mathbf{B}$.

iv If the difference between the new and old $\mathbf{e}$ exceeds a tolerance, return to (i). Else, put the solution in the unconstrained set $\hat{\mathcal{B}}$ if $\det(\mathbf{B}) > \bar{b}$.

v If the solution satisfies the winnowing constraints, put it in $\bar{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$.

Several points about the implementation of this approach bear discussion.

First, note that the shocks are constructed as $\mathbf{e} = \mathbf{B}^{-1}\boldsymbol{\eta}$ and require $\mathbf{B}^{-1}$ to be well behaved. For this reason we keep only solutions that satisfy a lower bound for $\det(\mathbf{B}) \geq \bar{b}$.

Second, we consider a large number of randomly chosen starting values, or initial guesses, for $\boldsymbol{\beta}$. Specifically, we initialize $\mathbf{B}$ to be the lower Cholesky factorization of $\boldsymbol{\Omega}$ for an arbitrary ordering of the variables (e.g., $(U_{Mt}, Y_t, U_{Ft})'$). We then rotate it by 40,000 random orthogonal matrices to give 40,000 initial guesses on $\mathbf{B}$ and hence the shocks. Completely random starting values will always deliver some $\mathbf{Z}(\boldsymbol{\beta})$ and $\mathbf{e}(\boldsymbol{\beta})$ with properties that are at odds with reasonable economic judgement. Our winnowing constraints are designed to exclude such solutions from $\overline{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$. We also estimate the model by GMM to verify that for a given initial guess, the solution agrees with the one obtained by IPEV estimation.

Third, the parameter values $\bar{c}$, $\overline{C}$, and $\bar{k}$ will in general vary with the data under investigation. For some choice of S, it is entirely possible that there exists no solution satisfying particular thresholds values. For the applications here, we set the $\bar{c}$, $\overline{C}$ to be relatively unrestrictive, with $\bar{c} = 0.03$ for the individual correlation, and $\overline{C} = 0.24$ for the collective correlation. The latter value corresponds to an average value of approximately 0.14 for the root-mean-square-correlation $\sqrt{\frac{1}{3}c(\boldsymbol{\beta})'c(\boldsymbol{\beta})}$. This says that a lower bound of 3% absolute correlation between stock market returns and both types of uncertainty shocks is maintained, with an average absolute correlation of 14%. For the presentation of results below we also show the single solution in the restricted set $\bar{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})$ with the highest collective correlation $\sqrt{c(\boldsymbol{\beta})'c(\boldsymbol{\beta})}$:

$$\hat{\boldsymbol{\beta}}^{\mathrm{maxC}} = \underset{\boldsymbol{\beta} \in \overline{\mathcal{B}}(\bar{c}, \bar{C}, \bar{k})}{\arg\max} \sqrt{c(\boldsymbol{\beta})'c(\boldsymbol{\beta})}. \tag{10}$$

Fourth, we set the parameters of the event constraints to $\bar{k}_1 = 4.0$, $\bar{k}_2 = 4.0$, and $\bar{k}_3 = 2$. The $\bar{k}_1$ and $\bar{k}_2$ thresholds pertain to the financial uncertainty shocks in October of 1987 when Black Monday occurred, and during the months of the 2007-09 financial crisis. This imposes the constraint that these events were accompanied by large financial uncertainty shocks. The requirement that the shocks be at least four standard deviations larger than the mean is guided by Bloom (2009). In his work, uncertainty shocks are calibrated from innovations to the VXO stock market volatility index. Bloom (2009) studies the dynamic effects of four standard deviation shocks to uncertainty. The $\bar{k}_3$ threshold states that the identified real activity shock cannot be too positive in the Great Recession; specifically, we restrict the shock to be no larger than two standard deviations above its sample mean in the Great Recession months.

To summarize, identification is predicated on three economic assumptions. First, components of the external S_t variables must exist that satisfy a minimum degree of non-zero correlation with the relevant set of uncertainty shocks ($\{e_{Mt}, e_{Ft}\}$ or $\{e_{Ft}\}$) and be exogenous with respect to the remaining structural shocks in $\mathbf{e}_t$ that form the compliment of this set. Second, the identified shocks must be consistent with a small number of extraordinary events whose interpretation is relatively incontrovertible. Third, purely idiosyncratic shocks to S_t are presumed not to affect the variables in $\mathbf{X}_t$ either contemporaneously or with a lag, an assumption that is tantamount to presuming that S_t can be excluded from the VAR. Below we show how this last assumption can be empirically evaluated.

It is worth noting that, in this application, the event constraints alone eliminate 99% of the solutions in $\hat{\mathcal{B}}$. When combined with lower bounds for the absolute correlations between the S_t and the uncertainty shocks, the qualitative nature of the solutions to the IPEV and GMM estimation problem is not found to be sensitive to starting values.

To have confidence in this implementation, Ludvigson, Ma, and Ng (2016) use Monte Carlo experiments to study the properties of the estimator. In general, the restrictiveness of correlation and event constraints required for precise identification varies with the data generating process (DGP). But the results for a DGP calibrated to the empirical application here suggest that the procedure produces solution sets that deliver the fairly narrow IRF bands around the true structural shocks and $\mathbf{B}$ matrix for the same functions for responses to shocks that display these properties in the historical sample, when the external S_t have properties consistent with observed values of $c(\boldsymbol{\beta})$, and when finite samples are set to those of the size used in this study.

3.3 System Estimation

The estimation procedure just discussed is based on an SVAR for $\mathbf{X}_t$. While S_t is used as part of an identification scheme, it is excluded from the SVAR. We refer to the foregoing analysis as the *subsystem approach*. In this section, we show that our approach can be fit into the classic simultaneous equations framework and interpreted as the output of a restricted system estimation for a larger VAR for $(\mathbf{X}_t, S_t)'$ with explicit restrictions on the structure of S_t. We refer to this as the *full system approach*. For this purpose, we consider a single S_t.

The full system VAR takes the same form as (1); the only difference is that S_t is now included in the VAR. The reduced form errors for the full system are $\boldsymbol{\eta}_t = (\eta'_{Xt}, \eta_{St})'$. The structural shocks are $\begin{pmatrix}\mathbf{e}'_{Xt} & e_{St}\end{pmatrix}'$ with $\boldsymbol{\eta}_t = \mathbf{B}\mathbf{e}_t$. The $\mathbf{B}$ matrix now has 16 parameters and the covariance structure gives 10 pieces of information. We assume that the shocks e_{St} do not contemporaneously affect $\mathbf{X}_t$. This means that the impact sub-vector giving the effects of e_{St}

on $\mathbf{X}_t$, denoted $\mathbf{B}_{XS} = (B_{MS}, B_{YS}, B_{FS})'$, is zero. These three zero restrictions imply

$$\begin{pmatrix} \eta_{Mt} \\ \eta_{Yt} \\ \eta_{Ft} \\ \eta_{St} \end{pmatrix} = \begin{pmatrix} B_{MM} & B_{MY} & B_{MF} & 0 \\ B_{YM} & B_{YY} & B_{YF} & 0 \\ B_{FM} & B_{FY} & B_{FF} & 0 \\ B_{SM} & B_{SY} & B_{SF} & B_{SS} \end{pmatrix} \begin{pmatrix} e_{Mt} \\ e_{Yt} \\ e_{Ft} \\ e_{St} \end{pmatrix}. \tag{11}$$

The synthetic variables $\mathbf{Z}_t$ are now defined as

$$\begin{aligned} Z_{1t} &= \eta_{St} - B_{SY}e_{Yt} = B_{SM}e_{Mt} + B_{SF}e_{Ft} + B_{SS}e_{St} \\ Z_{2t} &= Z_{1t} - B_{SM}e_{Mt} = B_{SF}e_{Ft} + B_{SS}e_{St}. \end{aligned}$$

Hence they are functions of the structural parameters. This treatment of $\mathbf{Z}_t$ is conceptually distinct from the subsystem analysis earlier when $\mathbf{Z}_t$ was treated as a residual from a projection. The full system analysis also requires the synthetic variables $\mathbf{Z}$ to satisfy the exogeneity restrictions of Assumption A, $E[Z_{1t}e_{Yt}] = E[Z_{2t}e_{Yt}] = E[Z_{2t}e_{Mt}] = 0$. These full system restrictions imply the same equations for solving the model as those used in the subsystem analysis, except that the residual η_{St} is used to construct $\mathbf{Z}_t$ in place of S_t. Estimation of the full system $(\mathbf{X}_t, S_t)'$ therefore proceeds exactly as in the GMM estimation of the subsystem. Given the block structure of $\mathbf{B}$, we can also use IPEV to solve the $\mathbf{X}$ subsystem and the S equation iteratively.

But as is the case with the subsystem analysis, the model is underidentified. The problem from the full system perspective is that Assumption A does not provide restrictions for B_{SM}, B_{SY}, B_{SF}. A large number of solutions can be consistent with the covariance structure of $\boldsymbol{\eta}_t$ and yet satisfy Assumption A. To address this problem, we again use Assumption B to help tie down these parameters. In the full system, the correlations used in the correlation constraints of Assumption B are given by

$$\begin{aligned} c_{1M}(\boldsymbol{\beta}) &= \frac{\text{corr}(Z_{1t}, e_{Mt})}{\sigma_{Z_1}} = \frac{B_{SM}}{\sqrt{B_{SM}^2 + B_{SF}^2 + B_{SS}^2}} \\ c_{1F}(\boldsymbol{\beta}) &= \frac{\text{corr}(Z_{1t}, e_{Ft})}{\sigma_{Z_1}} = \frac{B_{SF}}{\sqrt{B_{SM}^2 + B_{SF}^2 + B_{SS}^2}} \\ c_{2F}(\boldsymbol{\beta}) &= \frac{\text{corr}(Z_{2t}, e_{Ft})}{\sigma_{Z_2}} = \frac{B_{SF}}{\sqrt{B_{SF}^2 + B_{SS}^2}}, \end{aligned}$$

where the second equalities follow by recalling that e_{Mt} and e_{Ft} have unit standard deviations. Evidently, these correlations explicitly depend on the parameters of the S equation. Thus they are not invariant to orthonormal rotation of $\mathbf{e}_X$ and the parameters of the subsystem. Still, as in the subsystem analysis, requiring that $c_{1M}(\boldsymbol{\beta}) > \bar{c}, c_{1F}(\boldsymbol{\beta}) > \bar{c}, c_{2F} > \bar{c}$ and $c(\boldsymbol{\beta}) > \overline{C}$ may not be enough to point identify $\mathbf{B}$. We further reduce the number of admissible solutions by requiring that the event constraints of Assumption B hold for the stock market crash of 1987 and the Great Recession/financial crisis of 2008-09, as in the subsystem analysis.

It is of interest to compare the full and subsystem analysis. The full system estimation is in some ways more restrictive than the subsystem approach. In the subsystem analysis, the process that generates S_t is left unspecified. As such, it can be a function of variables other than $\mathbf{X}_t$, both contemporaneously, and at lags. By contrast, the full system approach specifies the process for S_t. Any misspecification in one equation can affect all equations in the system. On the other hand, the full system merely constrains the contemporaneous effect of S_t on $\mathbf{X}_t$ to zero. This is a weaker than assuming that S_t is exogenous for $\mathbf{X}_t$, which additionally prevents the lags of S_t from affecting $\mathbf{X}_t$. Constraining the current and lagged values of S_t to zero amounts to the subsystem analysis of excluding S_t from the larger VAR altogether. It should however be noted that excluding the past values of S_t from the equations for $\mathbf{X}_t$ is not needed for identification. Thus the assumption that S_t can be excluded from the VAR for $\mathbf{X}_t$ places overidentifying restrictions on the full system that can be evaluated empirically. A simple way to do so is to compare the impulse response functions estimated for the three variable system $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$ with those from a larger system that includes S_t but does not restrict the coefficients of S_{t-j} in the equations for $\mathbf{X}_t$ to zero, for $j \geq 1$. Denote these coefficients by $\mathbf{A}_{XS,j}$. We present these results below.

3.4 Measuring Uncertainty and Stock Market Returns

In our estimation we work with several different aggregate measures of uncertainty, which are indexes constructed over individual uncertainties for a large number of observable time-series. A long-standing difficulty with empirical research on this topic has been the measurement of uncertainty. JLN find that common uncertainty proxies contain economically large components of their variability that do not appear to be generated by a movement in genuine uncertainty across the broader economy. This occurs both because these proxies over-weight certain series in the measurement of aggregate uncertainty, and because they erroneously attribute forecastable fluctuations to a movement in uncertainty. Equity market volatility, for example, contains a non-trivial component generated from forecastable variation in stock returns. The estimated macro uncertainty index constructed in JLN is designed to address these issues and improve the measurement of aggregate uncertainty. The methodology used here for constructing uncertainty indexes follows JLN and we refer the reader to that paper for details.

Let $y_{jt}^C \in Y_t^C = (y_{1t}^C, \ldots, y_{N_C t}^C)'$ be a variable in category C. Its h-period ahead uncertainty, denoted by $\mathcal{U}_{jt}^C(h)$, is defined to be the volatility of the purely unforecastable component of the future value of the series, conditional on all information available. Specifically,

$$\mathcal{U}_{jt}^C(h) \equiv \sqrt{\mathbb{E}\left[(y_{jt+h}^C - \mathbb{E}[y_{jt+h}^C|I_t])^2|I_t\right]} \tag{12}$$

where I_t is information available. If the expectation today of the squared error in forecasting

y_{jt+h} rises, uncertainty in the variable increases. As in JLN, the conditional expectation of squared forecast errors in (12) is computed from a stochastic volatility model, while the conditional expectation $\mathbb{E}[y^C_{jt+h}|I_t]$ is replaced by a diffusion index forecast, augmented to allow for nonlinearities. These are predictions of an autoregression augmented with a small number of common factors $q_t = (q_{1t}, \ldots, q_{rt})'$ estimated from a large number of economic time series x_{it} each with factor representation $x_{it} = \Lambda'_{it} q_t + e_{\mathcal{X},it}$. Nonlinearities are accommodated by including polynomial terms in the factors, and factors estimated squares of the raw data. The use of large datasets reduces the possibility of biases that arise when relevant predictive information is ignored. Let $Y^C_t = (y^C_{1t}, \ldots, y^C_{N_C t})'$ generically denote the series that we wish to compute uncertainty in.

Uncertainty in category C is an aggregate of individual uncertainty series in the category :

$$U_{Ct}(h) \equiv \text{plim}_{N_C \to \infty} \sum_{j=1}^{N_C} \frac{1}{N_C} \mathcal{U}^C_{jt}(h) \equiv \mathbb{E}_C[\mathcal{U}^C_{jt}(h)]. \tag{13}$$

In this paper, we consider four categories of uncertainty:

Category (C)	Y^C_t	N_C
(M): Macro	all variables in χ^M	134
(F): Financial	all variables in χ^F	147
(R): Real activity	real activity variables in χ^M	73

We use two datasets covering the sample 1960:07-2015:04.[7] The first is a monthly *macro dataset*, $\mathcal{X}^M_t$, consisting of 134 mostly macroeconomic time series take from McCracken and Ng (2016). The second is a *financial dataset* $\mathcal{X}^F_t$ consisting of a 147 of monthly financial indicators, also used in Ludvigson and Ng (2007) and JLN, but updated to the longer sample. The real uncertainty index U_{Rt} is an equally-weighted average of the individual uncertainties about 73 series in Groups 1 through 4 of $\mathcal{X}^M$. These include output and income variables, labor market measures, housing market indicators, and orders and inventories. Additional predictors for variables in $\mathcal{X}^M_{it}$ include factors formed from $\mathcal{X}^F_{it}$ and vice-versa, squares of the first factor of each, and factors in the squares of individual series, $\left(\mathcal{X}^M_{it}\right)^2$ and $\left(\mathcal{X}^F_{it}\right)^2$.

Our use of stock returns S_t to generate instruments is grounded in the theoretical premise that both macro and financial uncertainty shocks should be reflected in stock market returns. There is no reason, however, that the regressands in (9a) and (9b) must be exactly the same measure of stock market activity. All measures of stock market activity are highly correlated because they contain a large common component (much of which is orthogonal to the rest of the economy). In order to introduce some additional independent variation in our two instruments, our base cases use different measures of aggregate stock market activity S_{1t} and S_{2t}, although in

[7] A detailed description of the series is given in the Data Appendix of the online location where updated JLN uncertainty index data are posted: http://www.sydneyludvigson.com/s/jln_data_appendix_update.pdf

practice we get very similar results if we use the same value-weighted stock market index return in (9a) and (9b). Specifically, for S_{1t}, the regressand for (9b), we use the Standard and Poor 500 stock market index return. For S_{2t}, the regressand in (9a), we use $\alpha_p \text{crsp}_t + (1-\alpha_p)\,\text{small}_t$, which is a portfolio weighted average of the return on the CRSP value-weighted stock index in excess of the one-month Treasury bill rate and the smallest decile stock market return in the NYSE.[8] We set the portfolio weight α_p to be a value close to one, thereby giving only a small amount of additional weight to small stocks. Small stocks are less representative of the market as a whole, and it is unclear how highly correlated they should be with aggregate uncertainty measures. For the base case results presented below we set $\alpha_p = 0.94$. However, we also investigated a range of values for $\alpha_p \in [0.75, 1]$ and found very similar estimates and impulse responses for all weights in this range, including setting $\alpha_p = 0.75$ and unity.

The parameters to be estimated include the reduced form VAR parameters in (2), from which we obtain $\hat{\boldsymbol{\eta}}_t$, the parameters in (9a) and (9b), from which we construct $Z_{1t}(\boldsymbol{\beta})$ and $Z_{2t}(\boldsymbol{\beta})$, the covariances between $Z_{1t}(\boldsymbol{\beta})$ and $Z_{2t}(\boldsymbol{\beta})$ and $\hat{\boldsymbol{\eta}}_t$, and the structural parameters using results from the preceding three estimations. The sample moment conditions can be collected into one vector and Generalized Method of Moments (GMM, Hansen (1982)) applied to estimate all parameters, where the number of moments equals the number of parameters. Serial correlation and heteroskedasticity robust standard errors are constructed as in Newey and West (1987) wherever asymptotic standard errors are reported for a single solution.

4 Results for $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$

This section presents empirical results. We begin by studying systems with macro uncertainty U_{Mt}. We then move on to consider real uncertainty U_{Rt} formed exclusively from real activity variables.

Our first VAR is defined by $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$. We consider $h = 1$ (one-month uncertainty) and several measures of Y_t. The first two measures are the log of real industrial production, denoted ip_t, and the log of employment, denoted emp_t. While industrial production is a widely watched economic indicator of business cycles, it only captures goods-producing industries and has been a declining share of GDP. Employment only covers the labor market. Hence we also consider an additional measure of real activity: the cumulated sum of the first common factor estimated from the macro dataset χ^M (since the raw data used to form this factor q_{1t} are transformed to stationary), which we denote Q_{1t}. Since our emphasis is on $h = 1$, we write U_{Mt} instead of $U_{Mt}(1)$, and analogously for U_{Ft}, in order to simplify notation. We refer to the estimation using data on $\mathbf{X}_t = (U_{Mt}, ip_t, U_{Ft})'$ as our *baseline case*.

The top panel of Figure 1 plots the estimated macro uncertainty U_{Mt} in standardized units

[8]The CRSP index is a value-weighted return of all stocks in NYSE, AMEX, and NASDAQ.

along with the NBER recession dates. The horizontal bar corresponds to 1.65 standard deviation above unconditional mean of each series (which is standardized to zero). As is known from JLN, the macro uncertainty index is strongly countercyclical, and exhibits large spikes in the deepest recessions. The updated data U_{Mt} series shows much the same. Though U_{Mt} exceeds 1.65 standard deviations 48 times, they are clustered around the 1973-74 and 1981-82 recessions, as well as the Great Recession of 2007-09. Macroeconomic uncertainty is countercyclical and has a correlation of -0.65 with the 12-month moving-average of the growth in industrial production.

The bottom panel of Figure 1 plots the financial uncertainty series U_{Ft} over time, which is new to this paper. U_{Ft} is a broad-based measure of time varying financial uncertainty using data from the bond market, stock market portfolio returns, and commodity markets. Hence, it is smoother than proxies such as VIX or any particular bond index. As seen from Figure 1, U_{Ft} is also countercyclical, though less so than U_{Mt}; the correlation with industrial production is -0.39. The series often exhibits spikes around the times when U_{Mt} are high. However, U_{Ft} is more volatile and spikes more frequently outside of recessions, the most notable being the 1987 stock market crash. Though observations on U_{Ft} exceed the 1.65 standard deviation line 33 times, they are spread out in seven episodes, with the 2008 and 1987 episodes being the most pronounced.

As is clear from Figure 1, both indicators of macro and financial uncertainty are serially correlated and hence predictable. They have comovements but also have independent variations as the correlation between them is 0.58. However, this unconditional correlation cannot be given a structural interpretation. The heightened uncertainty measures can be endogenous responses to events that are expected to happen, but they can also be exogenous innovations. We use a VAR to capture the predictable variations, and then identify uncertainty shocks from the VAR residuals using the restrictions described in the previous section.

4.1 SVAR Estimates and Uncertainty Shocks

Several features of the VAR estimates are qualitatively similar for all measures of Y_t. Table 1 highlights some of these results. For the purposes of this table, we show estimates for the single max C solution in (10). Panel A of this table shows that the sample correlation coefficients between Z_{1t} and $\hat{e}_{Mt}$ and $\hat{e}_{Ft}$, and between Z_{2t} and $\hat{e}_{Ft}$ are statistically significant and negative in each case, indicating that uncertainty shocks of both types tend to be high when stock market returns are low. Panel A also shows that the correlation between Z_{1t} and $\hat{e}_{Yt}$, and the correlation between Z_{2t} and $\hat{e}_{Yt}$ and $\hat{e}_{Mt}$ are all zero, which is true by construction of the algorithm and solution for $\mathbf{B}$. Panel B shows that σ_{MM}, σ_{YY}, and σ_{FF} are all strongly statistically significantly different from zero. This in turn indicates the presence of both macro and financial uncertainty shocks in the SVAR, as well as real activity shocks. Since both U_{Mt} and U_{Ft} are serially correlated, we should therefore find that Z_{1t} is correlated with lags of U_{Mt} and U_{Ft}, while Z_{2t} is correlated with lags of U_{Ft}. Results not reported confirm this is the case.

Figure 2 presents the time series of the standardized shocks (e_M, e_{ip}, e_F) identified from the system with $Y_t = ip_t$, again for the max C solution. All shocks display strong departures from normality with excess skewness and/or excess kurtosis. The largest of the e_{ip} shocks is recorded in 1980:04, followed by 1974:11, and 2005:09. There also appears to be a moderation in the volatility of the ip shocks in the post-1983 period. The largest macro uncertainty shock is in 1970:12, followed by the shock in 2008:10. The largest financial uncertainty shock is recorded in 1987:10 (Black Monday), followed by the shock in 2008:09 during the financial crisis. For e_F, the extreme but transitory nature of the 1987 stock market crash leads to a very large spike upward in e_F in the month of the crash, followed by a very large spike downward in the month following the crash as the market recovered strongly and quickly. While this episode magnifies the spike in e_F in 1987, it is largely orthogonal to real activity and macro uncertainty.

Observe that the large ip shock in 2005:09 is not associated with a contemporaneous spike in uncertainty, while there are several spikes in both types of uncertainty that do not coincide with spikes in e_{ip}. The next subsection uses impulse response functions to better understand the dynamic causal effects and propagating mechanisms of these shocks.

4.2 The Dynamic Effects of Uncertainty Shocks

Impulse response functions (IRFs) trace out the effects of counterfactual increases in the shocks. All plots show responses to one standard deviation changes in ϵ_{jt} in the direction that leads to an *increase* in its own variable X_{jt}.

Figure 3 shows in shaded areas the set of dynamic responses that satisfy the winnowing constraints for each variable in the SVAR to each structural shock for system with $Y_t = ip_t$. The dotted line shows the max C solution. Figure 4 displays the analogous plots for systems that use emp_t, and the real activity index Q_{1t}. We discuss the results displayed in both figures simultaneously.

The figures show that positive shocks to *financial* uncertainty e_F are found to lead to sharp declines in all three measures of real activity that persists for many months (center plot, bottom row). In all estimations, the sets of solutions that satisfy the identification restrictions in Assumption B have this pattern. These results lend support to the hypothesis that heightened financial uncertainty is an exogenous impulse that causes declines in real activity. Note, however, there is little evidence that high financial uncertainty is a consequence of lower economic activity. Instead, exogenous (positive) shocks to real activity either *increase* financial uncertainty or have no clear affect on it.

Positive perturbations to e_{Ft} also cause U_{Mt} to increase sharply. However, there is less evidence that shocks to macro uncertainty have effects on financial uncertainty: the set of solutions show positive response of financial uncertainty for the system with $Y = ip$, but the responses for the other two measures of real activity range from positive to zero to negative.

While we find no evidence that high financial uncertainty is a consequence of lower economic activity, the results for macro uncertainty are quite different. Both figures show that macro uncertainty falls sharply in response to positive real activity shocks. Alternatively stated, negative real activity shocks increase macro uncertainty sharply. These endogenous movements in macro uncertainty persist for well over a year after the real activity shock. This result is strongly apparent in all the solutions of the identified sets for Y measured as production or employment, suggesting that higher macro uncertainty in recessions is a direct endogenous response to lower economic activity. The responses in the system using the real activity index Q_{1t} as a measure of Y are inconclusive, as the identified set in this case includes a wide range surrounding zero even though the max C solution shows that U_M falls sharply in response to a positive Q_1 shock. However, there is no evidence that the observed negative correlation between macro uncertainty and real activity is driven by causality running in the opposite direction. Indeed, the top middle panels of each figure show that positive macro uncertainty shocks often *increase* real activity in the short run, consistent with growth options theories discussed above. The exception is again the system with $Y = Q_1$where the identified set displays a wide range of responses. But even in this case the max C solution shows the real activity index increases sharply after a positive shock to macro uncertainty.

4.3 The Structural Shocks and Decomposition of Variance

In Figure 1 presented earlier, we find 1973-74, 1981-82, and 2007-2009 to be the three episodes of heightened macroeconomic uncertainty, defined as the periods when U_{Mt} is at least 1.65 standard deviations above its unconditional mean. We now look for the "large adverse" shocks in the systems $(U_{Mt}, Y_t, U_{Ft})'$, with $Y_t = ip_t$, emp_t , Q_{1t}. More precisely, we consider large positive uncertainty shocks and large negative real activity shocks.

For the max C solution, Figure 5 displays the date and size of shocks that are at least two standard deviations above the mean, estimated using the three different measures of Y_t. In view of the non-normality of the shocks, the figure also plots horizontal lines corresponding to three standard deviation of the unit shocks, which is used as the reference point for 'large'. The lowest panel shows that, irrespective of the definition of Y_t, all SVARs identify big financial uncertainty shocks in October 1987 and in one or more months of 2008. Such solutions are selected as part of the identification scheme. The middle panel shows that large negative real activity shocks are in alignment with all post-war recessions with one exception: the negative real activity shock in 2005 is not immediately associated with a recession, but it could be the seed of the Great Recession that followed. It is known that the housing market led the 2007-2009 recession (e.g., see Favilukis, Ludvigson, and Van Nieuwerburgh (2015) for a discussion). Indeed, all 10 housing series in $\mathcal{X}^M$ (most pertaining to housing starts and permits series) exhibit sharp declines starting in September 2005 and continuing through 2006, thereby leading the Great Recession. This suggests that the negative spike in real activity in 2005 was partly driven by the housing sector.

The top panel of Figure 5 shows that the dates of large increases in e_M are less clustered. They generally coincide with, or occur shortly after, the big real activity shocks and the financial uncertainty shocks. Observe that large macro uncertainty shocks occurred more frequently in the pre-1983 than the post 1983 sample, consistent with a Great Moderation occurring over the period ending in the Great Recession. However, increases of greater than three standard deviations for e_M appear only when real activity is measured by production in the SVAR, a point we return to below.

To give a sense of the historical importance of these shocks, we perform a decomposition of variance, given by the fraction of s-step-ahead forecast error variance attributable to each structural shock ϵ_{Mt}, ϵ_{Yt}, and ϵ_{Ft} for $s = 1$, $s = 12$, $s = \infty$. We also report the maximum fraction of forecast error variance over all VAR forecast horizons s that is attributable to each shock, denoted $s = s_{max}$ in Table 2. Table 2 reports these results for the max C solutions in the system with $Y_t = ip_t$ (left column), $Y_t = emp_t$ (middle column), and $Y_t = Q_{1t}$ (right column).

According to the top row, two of the three real activity shocks, namely e_{ip}, and e_{Q_1}, have sizable effects on macroeconomic uncertainty U_M, with shocks to production explaining up to 67% of the variation in U_M. But according to the bottom row, these same shocks have small effects on financial uncertainty U_F. At the same time, positive macro uncertainty shocks e_M, which increase rather than decrease real activity, explain a surprisingly large fraction of production (up to 70%), employment (up to 20%) and the real activity index (up to 56%). On the other hand, financial uncertainty shocks e_F have a small contribution to the one-step-ahead forecast error variance of all three measures of real activity, but their relative importance increases over time. Financial uncertainty shocks explain up to 40% of the forecast error variance in production, up to 65% of the forecast error variance in employment, and up to 37% of the forecast error variance in the real activity index, compared to 31% for production. Financial uncertainty shocks e_F feed into U_M, and macroeconomic uncertainty shocks e_M also feed into U_F.

Regardless of which measure of real activity is used, we find that financial uncertainty is unlike macro uncertainty or real activity in that its variation is far more dominated by its own shocks. For example, in the system with ip, e_F shocks explain 97% of the $s = 1$ step-ahead forecast error variance in U_{Ft}, and 98% of the $s = \infty$ step-ahead forecast error variance. In the systems with *emp* and Q_1, e_F shocks explain 76% and 96%, respectively, of the $s = 1$ step-ahead forecast error variance in U_{Ft}, and 80% and 96%, respectively, of the $s = \infty$ step-ahead forecast error variance.

To summarize, in all three systems, real activity shocks e_Y have quantitatively large persistent negative effects on macro uncertainty U_M. In turn, macro uncertainty shocks e_M have large positive impact effects on real activity measures Y. Financial uncertainty shocks e_F have smaller impact effects but larger long run effects that dampen real activity Y. Across all systems, the forecast error variance of financial uncertainty is the least affected by shocks other than its own, suggesting that U_F is quantitatively the most important exogenous impulse in the system.

5 Uncertainty in Real Activity

The results discussed above suggest that the dynamic relationship between macro uncertainty and real activity can be quite different from the relation between financial uncertainty and real activity. However, given the composition of our data χ^M, macroeconomic uncertainty itself can be due to uncertainty in real activity variables such as output and unemployment, to price variables, and to financial market variables. The theoretical uncertainty literature has focused on modeling exogenous uncertainty shocks that arise specifically in measures of real economic fundamentals, rather than in prices or financial markets. To better evaluate the implications of these theoretical models, we consider systems that isolate uncertainty about real activity using the U_{Rt} sub-index that more closely corresponds to the theoretical literature.

5.1 System $\mathbf{X}_t = (U_{Rt}, Y_t, U_{Ft})'$

We isolate the real activity components of macro uncertainty by aggregating the individual uncertainty estimates over the 73 real activity variables in the macro dataset $\mathcal{X}^M$. The one-period ahead uncertainty in real activity, denoted U_{Rt}, is show in Figure 6. This series, like U_{Mt}, is countercyclical though somewhat less so, having a correlation of -0.50 with industrial production (as compared to -0.66 for U_{Mt}). At first glance, U_{Rt} appears to fluctuate in a manner similar to macroeconomic uncertainty U_{Mt}. The two series have a correlation of 0.71 and exhibit some overlapping spikes. But U_{Rt} and U_{Mt} also display notable independent variation. Figure 6 shows that there are 43 observations of U_{Rt} that are at least 1.65 standard deviations above its mean. These can be organized into five episodes: 1965, 1970, 1975, 1982-83, and 2007. By contrast, U_{Mt} in Figure 1 only exhibits three such episodes. Observe that the U_{Rt} series exhibits several spikes before 1970 that are not accompanied by spikes in U_{Mt}.

Given the distinctive patterns in the time series behavior of U_{Rt} and U_{Mt}, one might expect to find different dynamic relationships with the other variables in our systems when U_{Mt} is replaced by U_{Rt}. Surprisingly, the impulse responses functions are qualitatively similar to systems studied above that use broad-based macro uncertainty. The sets of responses that satisfy our winnowing constraints are displayed in Figure 7, with the dashed lines indicating the max C solution. The solid line for the $Y_t = ip_t$ system indicates that the set in that case is a singleton.

Figure 7 shows that, for all solutions that satisfy Assumption B and no matter which measure of real activity is used, (i) positive shocks to real activity measures unambiguously cause sharp declines in real economic uncertainty U_{Rt} so that negative shocks cause sharp increases in real economic uncertainty; (ii) positive real activity uncertainty shocks e_{Rt} do not cause declines in real activity measures; indeed the opposite is unambiguously true; (iii) positive financial uncertainty shocks e_{Ft} lead to sharp declines in real activity measures that are steep and persistent, and (iv) there is little evidence that high (low) financial uncertainty is caused by negative (positive) real activity shocks; the sets of IRFs surround zero for all systems. Thus the identified sets present an even clearer picture of the dynamic causal relationships in the systems with $\mathbf{X}_t = (U_{Rt}, Y_t, U_{Ft})'$ than they do in the systems with $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$.

Figure 8 plots the large adverse structural shocks for the max C solutions identified from the systems $(U_{Rt}, Y_t, U_{Ft})'$ for $Y_t = ip_t, emp_t, Q_{1t}$ analogous to Figure 2. The top panel shows that the real uncertainty shock e_{Rt} exhibits spikes in excess of three standard deviations during the Great Recession only for the system in which $Y_t = ip_t$. Moreover, for the other two systems in which $Y_t = emp_t$ or Q_{1t}, there is not a spike that exceeds even two standard deviations above its mean, despite the fact that U_{Rt} itself exhibits a large spike in the Great Recession (see Figure 6).

These episodes serve to reinforce the conclusion from the IRFs that the heightened real economic uncertainty in recessions is often endogenous response to other shocks, rather than an exogenous impulse. Even though there were many large spikes in real uncertainty shocks e_{Rt} pre-1983, there have been fewer large adverse shocks to real economic uncertainty since 1983, a period that coincides with the so-called Great Moderation.

To complete the analysis, we present variance decompositions for the system $(U_{Rt}, Y_t, U_{Ft})'$, with three measures of real activity $Y_t = ip_t, emp_t, Q_{1t}$. These results, presented in Table 3, share some similarities with the systems that use macro uncertainty U_{Mt} shown in Table 2, but there are at least two distinctions. First, financial uncertainty shocks decrease real activity and explain larger fractions of the forecast error variance in two measures of real activity at long horizons. At the longest $s = \infty$ VAR horizon, financial uncertainty shocks explain 62% of forecast error variance in employment and 58% of the forecast error variance in the real activity index. These results suggest that financial uncertainty has quantitatively large negative consequences for at least some measures of real activity. Second, compared to systems that use U_{Mt}, smaller fractions of the forecast error variance in U_{Rt} are explained by its own shocks for two of the three systems corresponding to $Y_t = ip_t, emp_t$. This is largely because, in these systems, shocks to ip_t and emp_t have larger effects on real activity uncertainty than on macro uncertainty. This reinforces the point that countercyclical increases in real economic uncertainty are best characterized as endogenous responses to declines in real activity.

6 Robustness and Additional Cases

This section presents results for a number of additional cases that help assess robustness of our results.

First, we present results when only event constraints or correlation constraints are imposed, but not both. Second, we test whether restrictions implied by recursive identification are supported by the data. Third, we consider an estimation where we presume two observed external variables are valid external instruments Z_{1t} and Z_{2t}, even though they may in fact contain an endogenous component. This is compared to the case where the same variables are presumed not to be valid instruments and IPEV is used to construct such instruments from S_{1t} and S_{2t}. Finally, we present results for the full system estimation described above for a VAR in $\mathbf{X}_t$ and S_t with explicit restrictions on the structure of S_t.

6.1 Eliminating Event or Correlation Constraints

Assumption B imposes two types of winnowing constraints, the event constraints and the correlation constraints. If either of these were not helpful in narrowing the solution sets, then failure to impose one or the other would not have a significant affect on the results. Figure 9 shows the dynamic responses when we impose either one or the other type constraint (but not both). The top panel shows the results imposing only event constraints, and the bottom panel shows the results imposing only correlation constraints. The identified sets of IRFs in both panels are noticeably wider, indicating that each type of constraint contributes to identification. However, the event-constraints-only panel shows that all solutions in the identified set imply that financial uncertainty shocks drive down emp_t and Q_{1t}, though the set for ip_t is too wide to draw such a conclusion. Many, though not all, of the solutions show that positive real activity shocks drive down macro uncertainty, whereas both positive and negative responses of real activity are equally well represented in reaction to macro uncertainty shocks.

A similar pattern emerges in the bottom panel, where results are reported for the case in which only correlation constraints are imposed. On the whole, the sets are at least as wide or wider around zero compared to the top panel, indicating that the event constraints are more important than the correlation constraints. Taken together, the results in both panels demonstrate the importance of both constraints for drawing clear conclusions about the dynamic causal effects in the system.

6.2 Tests of Recursive Identification Restrictions

The econometric model permits us to test whether a recursive structure is supported by the data. Specifically, the assumptions in our winnowing constraints do not rule out the possibility of a recursive structure. To carry out the test we focus on a particular solution in the identified set, namely the max C solution. This solution is not on or near the boundary of the permitted parameter space given by the winnowing constraints and we simply approach the problem as the point-identified one. Given that $\sqrt{T}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1^0)$ is asymptotically $N(0, \boldsymbol{\Sigma}^2_{\hat{\boldsymbol{\beta}}_1})$, the null hypothesis of a recursive structure is a test that the three components of $\boldsymbol{\beta}_1$ corresponding to the off-diagonal entries of $\mathbf{B}$ are jointly zero. Hence the Wald statistic is chi-square distributed with three degrees of freedom. We first confirm that the test has the correct size in Monte Carlo simulations. Our estimates based on historical data strongly reject a lower triangular $\mathbf{B}$ for *any* possible ordering of the variables. Table 4 shows results from Wald tests with $Y_t = ip_t$, and either using U_{Mt} (first column) or U_{Rt} (second column).

What happens to the dynamic responses when we nevertheless impose restrictions based on recursive identification (and freely estimate the rest of the parameters)? With these recursive restrictions the SVAR is exactly identified so no winnowing constraints are needed. Figure 10 shows the dynamic responses for the ordered system $\mathbf{X}_t = (U_{Ft}, U_{Mt}, ip_t)'$ with bootstrapped standard error bands in dashed lines. There are many possible recursive orderings, and inevitably, the estimated IRFs differ in some ways across these cases. However, the dynamic responses under recursive identification have one common feature that is invariant to the ordering. Under recursive identification, no matter the which ordering is presumed, macro uncertainty shocks appear to cause a sharp decline in real activity, much like financial uncertainty shocks, while positive real activity shocks have little effect on macro uncertainty in the short run and if anything increase it in the long run. This is in stark contrast to the results from our identification scheme. Recall that our identification scheme is capable of recovering a recursive structure if it were true, but we failed to find such a structure. Further investigation reveals that the SVARs we study display non-zero contemporaneous correlations between U_{Ft} and Y_t, as well as between U_{Mt} and Y_t, a finding that is inconsistent with any recursive ordering. This result is robust across any of the six possible recursive orderings.[9] These results show that imposing a structure that prohibits contemporaneous feedback may spuriously suggest that macro uncertainty shocks are a cause of declines in real activity, rather than an endogenous response. The finding underscores the challenges of relying on convenient timing assumptions to sort out cause and effect in the relationship between uncertainty and real activity.

[9] The figures for these cases are omitted to conserve space but are available upon request.

6.3 Observed Instruments Case

The traditional approach to identification when the variables are simultaneously determined relies on the existence of valid instruments Z that are exogenous and relevant. In many applications, few if any plausible instruments exist that satisfy these restrictions. We've argued above that applications that seek to identify the empirical effects of uncertainty on real activity (and vice versa) are likely to be among those for which valid observed instruments are hard to find or identify. It is nevertheless of interest to consider an estimation in which two observed external variables S_{1t} and S_{2t} are *presumed* to be valid external instruments Z_{1t} and Z_{2t} and treated as such, even though they may in fact contain an endogenous component. This approach to estimation may be compared to the IPEV approach, where these same external variables are suspected to be possibly invalid (or imperfect) instruments because they may contain an endogenous component. For this exercise, we take Z_{1t} and S_{1t} to be estimated uncertainty for the $S\&P$ 500 stock market index return, denoted U_{SPXt}, and Z_{2t} and S_{2t} to be CRSP value-weighted excess stock market return, denoted r_{CRSPt}.

To motivate the use of these two external variables as reasonable choices for the presumed valid instruments, we first employ the external variables $S_{1t} = U_{SPXt}$ and $S_{2t} = r_{CRSPt}$ to estimate the system $X_t = (U_{Mt}, ip_t, U_{Ft})'$ using the method of IPEV described above, imposing Assumption B and using the same quantitative bounds $\overline{c}$, $\overline{k}_1$, $\overline{k}_2$, and $\overline{k}_3$ as used previously. We then verify that, for the max C solution, the estimated slope coefficient in a regression of S_{1t} on $\hat{e}_{ipt}$ is not statistically different from zero, and the estimated slope coefficients in a multivariate regression of S_{2t} on $\hat{e}_{ipt}$ and $\hat{e}_{Mt}$ are also not statistically different from zero, thereby satisfying the exogeneity requirements of classic IV vis-a-vis the estimated shocks.[10] The statistical insignificance of the coefficients is at least suggestive that the external variables $S_{1t} = U_{SPXt}$ and $S_{2t} = r_{CRSPt}$ might credibly behave as valid external instruments Z_{1t} and Z_{2t}. We could call these "ex-post valid IVs."

When we presume, as in this case, that two external variables are valid instruments, we directly apply Assumption A setting $Z_{1t} = S_{1t}$ and $Z_{2t} = S_{2t}$. The solution for $\mathbf{B}$ then follows from Proposition 1 and can be obtained in closed form. No winnowing constraints are imposed and no projections are performed. This estimation may be directly compared to the analogous estimation where we do not presume these variables are valid IVs, and instead employ IPEV using $S_{1t} = U_{SPXt}$ and $S_{2t} = r_{CRSPt}$ following the method described above. Figure 11 shows both sets of dynamic responses along with bootstrapped 90% error bands as vertical lines for the base case system $X_t = (U_{Mt}, ip_t, U_{Ft})'$. The figure shows that, qualitatively, most dynamic responses are similar to those obtained above for the base case, and to each other. However, the bootstrap error bands tend to be far wider for the ex-post valid IV case than the IPEV case

[10] Both regressions control for one lag of the dependent variable.

max C solution, especially for the responses to macro uncertainty and real activity shocks.

In our experience, the bootstrap standard error bands in conventional IV estimation of the SVAR tend to be wide when the external instruments only weakly identify some elements of $\mathbf{B}$. By contrast, the IPEV case displays much narrower bootstrap error bands for most IRFs. This happens because the winnowing constraints bring more information to bear, thereby improving the efficiency of the estimates. This points to a potential advantage of IPEV over traditional IV, when plausible restrictions on the shocks can be imposed to improve efficiency.

Sampling uncertainty aside, it is notable that the qualitative nature of the responses in Figure 11 for both the ex-post valid IV and IPEV estimations are similar to that obtained for the baseline case above: positive financial uncertainty shocks drive down production sharply and persistently, while positive production shocks endogenously decrease macro uncertainty but not financial uncertainty. Likewise, there is no evidence that positive macro uncertainty shocks drive down production.

6.4 System Estimation Results

This section reports the results of estimating the model using the full system approach described above. We estimate a four variable system in $(\mathbf{X}_t, S_t)'$, imposing $\mathbf{B}_{XS} = \mathbf{0}$, but without imposing $\mathbf{A}_{XS,j} = \mathbf{0}$ for $j \geq 1$. We report results for the four variable case where S_t is measured as the return on the CRSP value-weighted stock market index. Figure 12 presents the set identified IRFs for the full system estimation, where we impose the same Assumption B winnowing constraints imposed for the subsystem analysis. In this estimation the identified set for the system with $Y = Q_1$ is a singleton, as indicated by the solid lines for the responses in that case. The figure shows that the results are qualitatively very similar to the subsystem case. As for that case, positive shocks to financial uncertainty drive down all measures of real activity sharply and persistently, but there is no evidence that positive shocks to macro uncertainty decrease real activity. Again the opposite is true. Positive shocks to real activity clearly drive down macro uncertainty, especially for the systems in which real activity is measured as production or the real activity index Q_{1t}.

As discussed above, the subsystem exclusion restriction for S_t places overidentifying restrictions on the full system estimation. A simple way to evaluate this restriction is to compare the impulse response functions estimated for the three variable subsystem for $\mathbf{X}_t = (U_{Mt}, Y_t, U_{Ft})'$ alone, with those from the larger system that includes S_t but does not restrict the coefficients of S_{t-j} in the equations for $\mathbf{X}_t$ to zero, for $j \geq 1$. We denote these coefficients by $\mathbf{A}_{XS,j}$. Figure 13 presents the sets of identified impulse responses that satisfy Assumption B for the both the full system and the subsystem estimations overlayed on one another. To avoid clutter in the figure, we present the responses only for the systems with $Y = ip$. As the figure shows, the identified sets lie almost on top of each other, indicating that the responses are little different. Indeed, the coefficients on lags of S_t appear to be close to zero in all three $\mathbf{X}_t$ equations. The data thus appear qualitatively consistent with the restrictions that $\mathbf{A}_{XS,j} = \mathbf{0}$ for $j \geq 1$ and therefore the assumption that stock returns can be excluded from the VAR for $\mathbf{X}_t$.

In summary, even though a VAR that directly incorporates S is possible, the system estimation approach restricts S to be explained only by lags of S and $\mathbf{X}$, which could in general be restrictive. Our the three variable approach is more robust to such misspecification that could affect the entire system. On the other hand, the system estimation allows lags of S_t to feed back into future $\mathbf{X}_t$ whereas they are restricted to have no impact in the subsystem approach. These form part of the exclusion restriction on S_t. Estimation of the four variable system that includes S_t suggests that these exclusion restrictions are qualitatively consistent with the data.

7 Conclusion

A growing body of research establishes uncertainty as a feature of deep recessions but leaves open two key questions: is uncertainty primarily a source of business cycle fluctuations or an endogenous response to them? And where does uncertainty originate? There is no theoretical consensus on the question of whether uncertainty is a cause or a consequence of declines in economic activity. In most theories, it is modeled either as a cause or an effect but not both, underscoring the extent to which the question is fundamentally an empirical matter.

The objective of this paper is to address both questions econometrically using small-scale structural VARs that are general enough to nest the range of theoretical possibilities in empirical tests. Commonly used recursive identification schemes cannot achieve this objective, since by construction they rule out the possibility that uncertainty and real activity could influence one another contemporaneously. The econometric model employed in this paper nests the recursive identification scheme, and we find that it is strongly rejected by the data. An empirical model in which uncertainty and real activity simultaneously influence each other fits the data far better than one in which these relationships are restricted by timing assumptions that prohibit contemporaneous feedback.

To identify dynamic causal effects, this paper takes an alternative identification approach that relies on covariance restrictions with key external variables, and timing of extraordinary economic events to constrain the set of credible structural parameters. Both aspects impose economic assumptions about the behavior of the structural shocks that allow sets of solutions to be identified. We call this approach iterative projection external variable (IPEV). In addition, our empirical analysis explicitly distinguishes macro uncertainty and uncertainty about real activity from financial uncertainty, thereby allowing us to shed light on the origins of uncertainty shocks that drive real activity lower, to the extent that any of them do. The econometric framework allows uncertainty to be an exogenous source of business cycle fluctuations, or an endogenous response to them, or any combination of the two, without restricting the timing of these relationships.

Estimates of the econometric model are used to inform the nature of these dynamic relationships in U.S. data. The results from these estimations show that sharply higher uncertainty about real economic activity in recessions is fully an endogenous response to business cycle fluctuations, while uncertainty about financial markets is a likely source of them. Exogenous declines in economic activity have quantitatively large effects that drive real economic uncertainty endogenously higher. Financial uncertainty, by contrast, is dominated by its own shocks, implying that it is primarily an exogenous impulse vis-a-vis real activity and macro uncertainty. These results reinforce the hypothesis laid out in much of theoretical uncertainty literature, namely that uncertainty shocks are a source of business cycle fluctuations. They also stand in contrast to this literature, which has emphasized the role of uncertainty fluctuations in productivity and other real economic fundamentals. The findings here imply that the uncertainty shocks that drive real activity lower appear to have their have origins, not in measures of real activity, but in financial markets.

Appendix

Closed-Form Solution for B when Z is observed

Lemma 2 *There exists a unique solution to the system (7) if* $\mathbb{E}\left[e_{Ft}Z_2\right] \neq 0$ *and* $\mathbb{E}\left[e_{Mt}Z_1\right] \neq 0$.

Proof. To facilitate the presentation throughout the proof, let

$$\begin{aligned}
\boldsymbol{\eta}_t &= \mathbf{B}\mathbf{e}_t \\
\mathbf{B} &= \left[\underset{3\times 1}{\mathbf{B}_M}, \underset{3\times 1}{\mathbf{B}_Y}, \underset{3\times 1}{\mathbf{B}_F}\right] \\
\boldsymbol{\Omega} &= \mathbb{E}\left(\boldsymbol{\eta}_t\boldsymbol{\eta}_t'\right).
\end{aligned}$$

Let $\phi_{1F} = c_{1F}\sigma_{Z1}$, $\phi_{2F} = c_{2F}\sigma_{Z2}$, $\phi_{1M} = c_{1M}\sigma_{Z1}$. We have two external instruments (Z_1, Z_2) satisfying

$$\begin{aligned}
\mathbb{E}\left[e_{Ft}Z_1\right] &\equiv \phi_{1F} \neq 0, \mathbb{E}\left[e_{Mt}Z_1\right] \equiv \phi_{1M} \neq 0 \text{ and } \mathbb{E}\left[e_{Yt}Z_1\right] = 0 \\
\mathbb{E}\left[e_{Ft}Z_2\right] &\equiv \phi_{2F} \neq 0 \text{ and } \mathbb{E}\left[e_{Mt}Z_2\right] = \mathbb{E}\left[e_{Yt}Z_2\right] = 0
\end{aligned}$$

Then

$$\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right] = \mathbb{E}\left[\mathbf{B}\mathbf{e}_t Z_2\right] = \mathbf{B}\begin{bmatrix} 0 \\ 0 \\ \phi_{2F} \end{bmatrix} = \phi_{2F}\mathbf{B}_F \qquad \textbf{(A.1)}$$

Thus $\mathbf{B}_F$ exists if $\phi_{2F} \neq 0$. Observe that, since

$$\boldsymbol{\Omega} = \mathbb{E}\left[\boldsymbol{\eta}_t\boldsymbol{\eta}_t'\right] = \mathbf{B}\mathbf{B}'$$

we have

$$\mathbf{B}'\boldsymbol{\Omega}^{-1}\mathbf{B} = \mathbf{I}$$

hence, $\forall i, j = M, Y, F$

$$\mathbf{B}_j'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_i = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}.$$

Therefore,

$$\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]' \Omega^{-1}\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right] = \left(\phi_{2F}\mathbf{B}_F\right)' \boldsymbol{\Omega}^{-\frac{1}{2}}\boldsymbol{\Omega}^{-\frac{1}{2}}\left(\phi_{2F}\mathbf{B}_F\right) = \phi_{2F}^2$$

This implies that the scale ϕ_{2F} is identified up to a sign by

$$\phi_{2F} = \pm\sqrt{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]\boldsymbol{\Omega}^{-1}\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}. \qquad \textbf{(A.2)}$$

Next,

$$\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right] = \mathbb{E}\left[\mathbf{B}\mathbf{e}_t Z_1\right] = \mathbf{B}\begin{bmatrix} \phi_{1M} \\ 0 \\ \phi_{1F} \end{bmatrix} = \phi_{1M}\mathbf{B}_M + \phi_{1F}\mathbf{B}_F$$

But note that

$$\begin{aligned}\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]\boldsymbol{\Omega}^{-1}\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right] &= \phi_{2F}\mathbf{B}_{F'}\boldsymbol{\Omega}^{-1}\left(\phi_{1M}\mathbf{B}_M+\phi_{1F}\mathbf{B}_F\right)\\ &= \phi_{2F}\mathbf{B}_{F'}\left(\mathbf{B}\mathbf{B}'\right)^{-1}\left(\phi_{1M}\mathbf{B}_M+\phi_{1F}\mathbf{B}_F\right)\\ &= \phi_{2F}\phi_{1F}\end{aligned}$$

This implies that ϕ_{1F} is identified as

$$\phi_{1F}=\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]\boldsymbol{\Omega}^{-1}\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]}{\phi_{2F}}$$

which in turn implies

$$\phi_{1M}\mathbf{B}_M=\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}}c_{1F}. \qquad (\mathbf{A.3})$$

Thus solution to $\mathbf{B}_M$ exists if $\phi_{1M}\neq 0$. Furthermore, note that

$$\begin{aligned}&\left(\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}^2}\phi_{1F}\right)'\Omega^{-1}\left(\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}^2}c_{1F}\right)\\ =\ &\boldsymbol{\Omega}^{-\frac{1}{2}}\mathbf{B}_M\phi_{1M}^2\mathbf{B}_M'\boldsymbol{\Omega}^{-\frac{1}{2}}=\phi_{1M}^2\end{aligned}$$

This implies that the parameter ϕ_{1M} is identified up to a sign as

$$\phi_{1M}^2=\left(\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}^2}c_{1F}\right)'\boldsymbol{\Omega}^{-1}\left(\mathbb{E}\left[\boldsymbol{\eta}_t Z_1\right]-\frac{\mathbb{E}\left[\boldsymbol{\eta}_t Z_2\right]}{\phi_{2F}^2}\phi_{1F}\right). \qquad (\mathbf{A.4})$$

It only remains to identify $\mathbf{B}_Y$. $\mathbf{B}_Y$ must satisfy

$$\begin{aligned}\mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_Y&=1\\ \mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_M&=0\\ \mathbf{B}_Y'\boldsymbol{\Omega}^{-1/2}\boldsymbol{\Omega}^{-1/2}\mathbf{B}_F&=0\end{aligned} \qquad (\mathbf{A.5})$$

$\mathbf{B}_Y$ can be solved analytically using ($\mathbf{A.5}$) provided that $\mathbf{B}_F$ and $\mathbf{B}_Y$ are identified. In addition, since the equation ($\mathbf{A.5}$) is quadratic in $\mathbf{B}_Y$, $\mathbf{B}_Y$ is unique up to sign. It follows that there exists a τ such that

$$\mathbf{B}_Y=\tau\tilde{\mathbf{B}}_Y \qquad (\mathbf{A.6})$$

where $\tilde{\mathbf{B}}_Y$ is unique conditional on ϕ_{2F} and ϕ_{1M}, but the scalar τ is unique up to sign.

This shows that the solution to the system (7) exists and is unique up to sign if $\phi_{2F}\neq 0$, $\phi_{1M}\neq 0$. Combined with unit effect normalization (5) and the restriction on the admissible parameter space (4), $\mathbf{B}$ can be uniquely identified. The unit effect normalization implies

$$\begin{aligned}\begin{pmatrix}B_{MM}&B_{MY}&B_{MF}\\ B_{YM}&B_{YY}&B_{YF}\\ B_{FM}&B_{FY}&B_{FF}\end{pmatrix}&=\begin{pmatrix}1&H_{MY}&H_{MF}\\ H_{YM}&1&H_{YF}\\ H_{FM}&H_{FY}&1\end{pmatrix}\begin{pmatrix}\sigma_{MM}&0&0\\ 0&\sigma_{YY}&0\\ 0&0&\sigma_{FF}\end{pmatrix}\\ &=\begin{pmatrix}\sigma_{MM}&H_{MY}\sigma_{YY}&H_{MF}\sigma_{FF}\\ H_{YM}\sigma_{MM}&\sigma_{YY}&H_{YF}\sigma_{FF}\\ H_{FM}\sigma_{MM}&H_{FY}\sigma_{YY}&\sigma_{FF}\end{pmatrix}\end{aligned}$$

Combined with the restriction $\sigma_{jj} > 0$ for all $j = M, Y, F$, implies $B_{jj} > 0$ for all $j = M, Y, F$. From equation (**A.1**), $B_{FF} > 0$ pins down the sign of ϕ_{2F} conditional $\mathbf{Z}_t$. Since the sign of ϕ_{2F} is pinned down, the signs of B_{MF} and B_{YF} are also pinned down by the same restriction. From equation (**A.3**), $B_{MM} > 0$ pins down the sign of ϕ_{1M} conditional $\mathbf{Z}_t$ and therefore the signs of B_{YM} and B_{FM} are pinned down by the same restriction. It only remains to show the uniqueness of $\mathbf{B}_Y$. Provided that $\mathbf{B}_F$ and $\mathbf{B}_Y$ are identified and given the closed-form solution (**A.5**) that is quadratic in $\mathbf{B}_Y$, then $B_{YY} > 0$ pins down the sign of τ conditional $\mathbf{Z}_t$ and hence the sign of B_{MY} and B_{FY} are also pinned down by the same restriction. ■

The system of equations defining $\mathbf{B}$ is

$$0 = \mathbb{E}[\mathbf{g}_1(\mathbf{m}_{1t}; \boldsymbol{\beta}_1)] \equiv \overline{\mathbf{g}}_1.$$

The rank condition is satisfied when $\mathbf{J} \equiv \partial \mathbb{E}_T[\mathbf{g}_1]/\partial \boldsymbol{\beta}_1'$ is full column rank. We check that the rank condition is satisfied by evaluating $\mathbf{J}$ at the estimated parameter values for each case.

Procedure for Bootstrap

The bootstrap follows Krinsky and Robb (1986). Let $\hat{\boldsymbol{\beta}}$ and $\hat{\Theta}$ be the estimated GMM parameters and covariance of parameters for each case. We sample repeatedly from the joint distribution $N\left(\hat{\boldsymbol{\beta}}, \hat{\Theta}/T\right)$, where $\hat{\Theta}$ is the estimated GMM variance-covariance matrix to obtain $\mathcal{B}$ new sets of parameters $\hat{\boldsymbol{\beta}}^{(1)}....\hat{\boldsymbol{\beta}}^{(B)}$. For each $\hat{\boldsymbol{\beta}}^{(i)}$ we infer the $\mathbf{e}^{(i)}$ for that draw and check that the winnowing constraints are satisfied. If they are, we keep the draw. If not, we redraw. We continue until the number of kept draws $\mathcal{B}$=10,000. From these $\mathcal{B}$ saved draws, we calculate the impulse response function values at each draw, $\boldsymbol{\Upsilon}_{s,j}^{(1)}, ..., \boldsymbol{\Upsilon}_{s,j}^{(B)}$, where s indexes the VAR horizon and j the variable being shocked, and where $\boldsymbol{\Upsilon}_{s,j}^{(b)} = \boldsymbol{\Upsilon}_{s,j}\left(\hat{\boldsymbol{\beta}}^{(b)}\right)$. The confidence intervals are ranges for $\boldsymbol{\Upsilon}_{s,j}^{(b)}$ created by trimming $\alpha/2$ from each tail of the resulting distribution of the function values.

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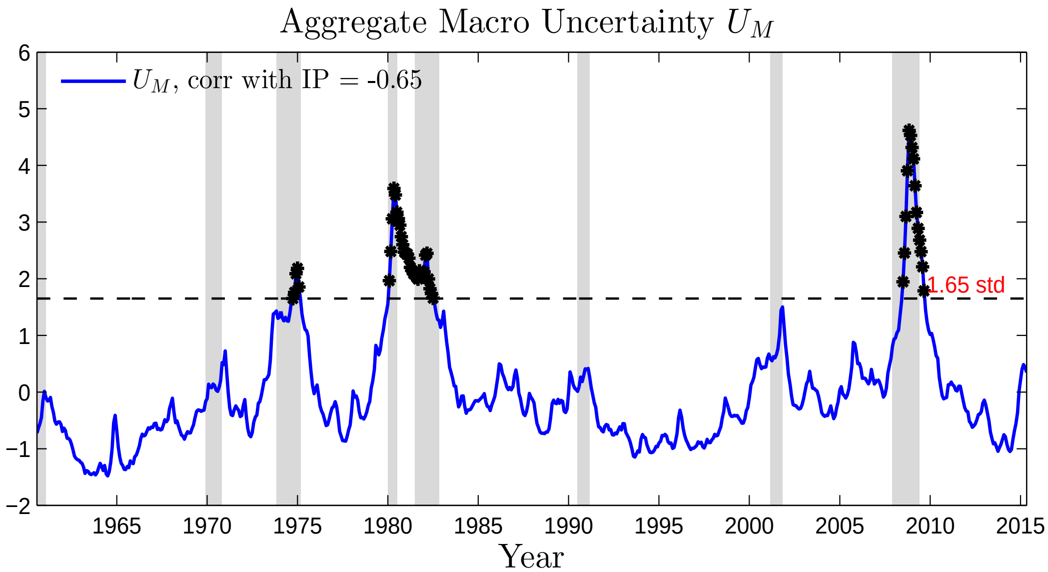
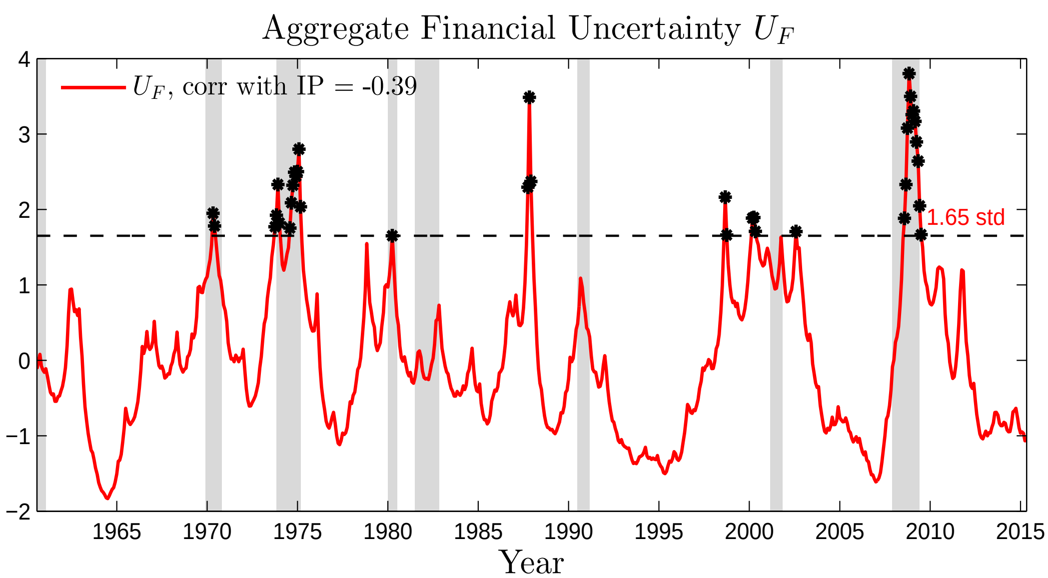
8 Figures and Tables

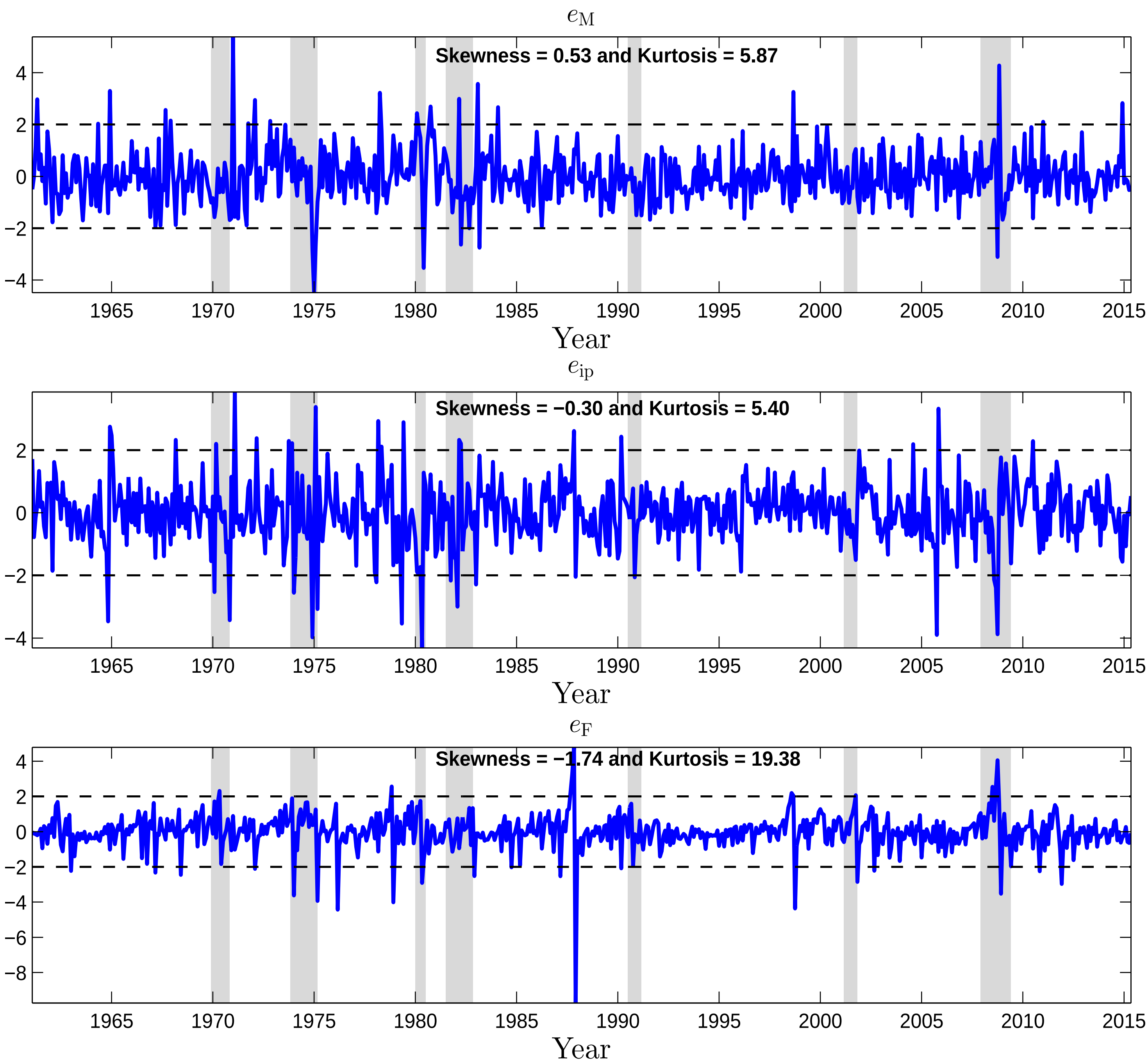
Figure 1: Macro and Financial Uncertainty Over Time





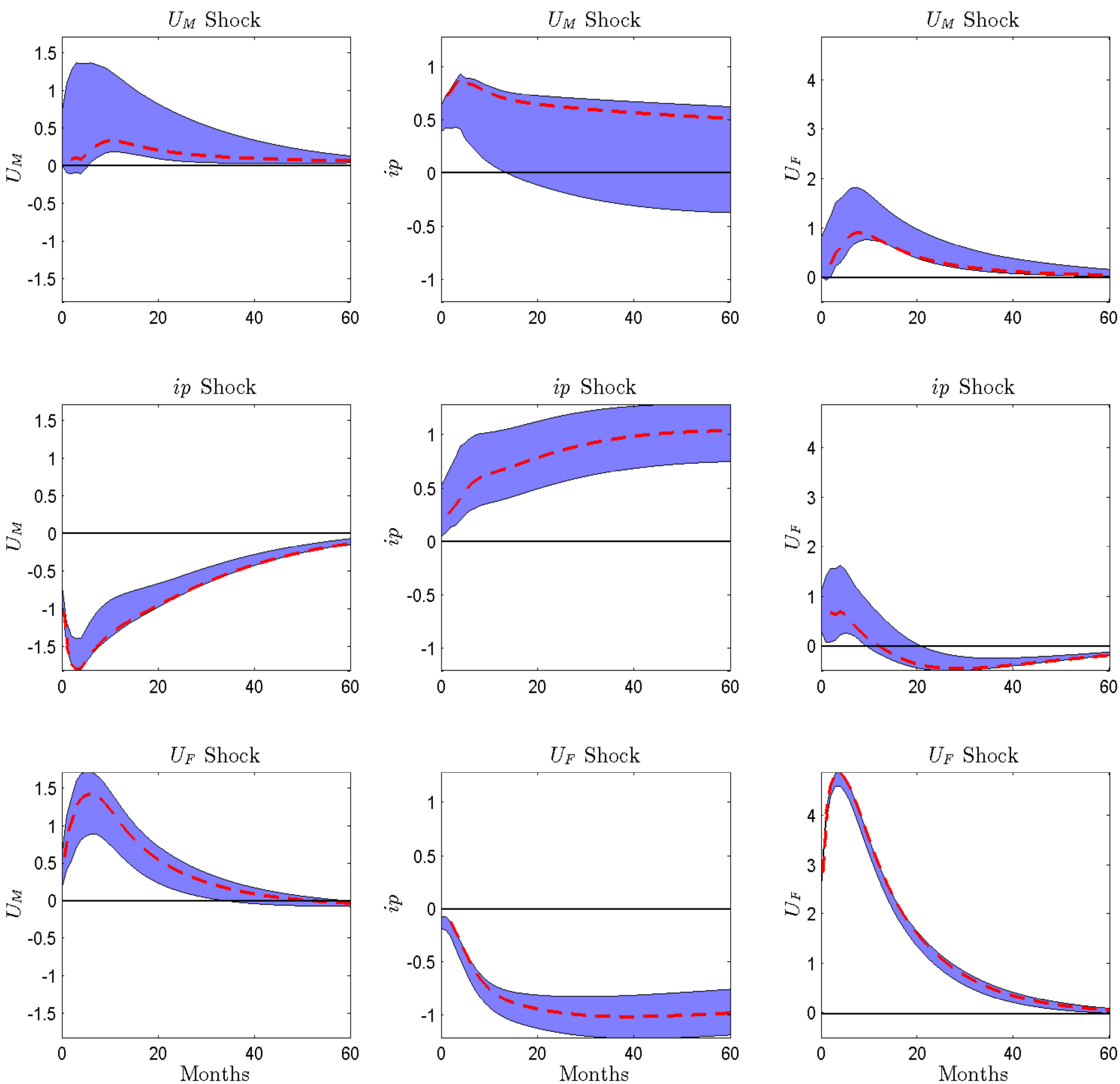
The upper panel plots the time series of the macro uncertainty U_M, expressed in standardized units. The lower panel shows the time series of financial uncertainty U_F expressed in standardized units. The shaded areas correspond to the NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when uncertainty is 1.65 standard deviations above its unconditional mean. The data are monthly and span the period 1960:07 to 2015:04.

Figure 2: Time Series of e Shock from SVAR System $(U_M, ip, U_F)'$



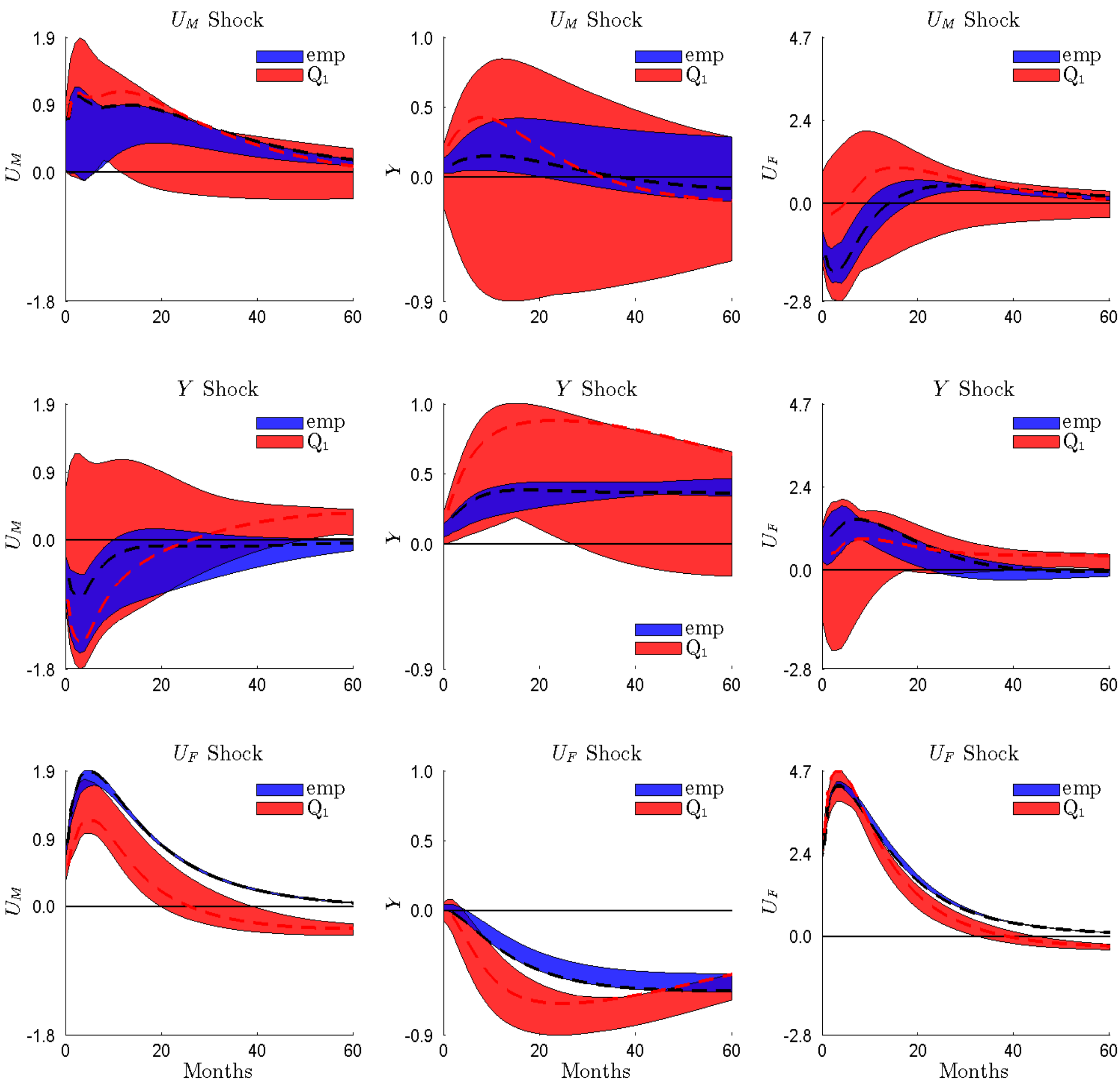
The horizontal line corresponds to 3 standard deviations above/below the unconditional mean of each series. The shocks $e = B^{-1}\eta_t$ for max C solution are reported, where η_t is the residual from VAR(6) of $(U_M, ip, U_F)'$ and $B = A^{-1}\Sigma^{\frac{1}{2}}$. Skewness is defined as $s = \frac{\sum_t^T (e_t - \bar{e})^3/T}{Var(e)}$. Kurtosis is defined as $\kappa = \frac{\sum_t^T (e_t - \bar{e})^4/T}{[Var(e)]^2}$. The sample spans the period 1960:07 to 2015:04.

Figure 3: IRFs of SVAR $(U_M, ip, U_F)'$



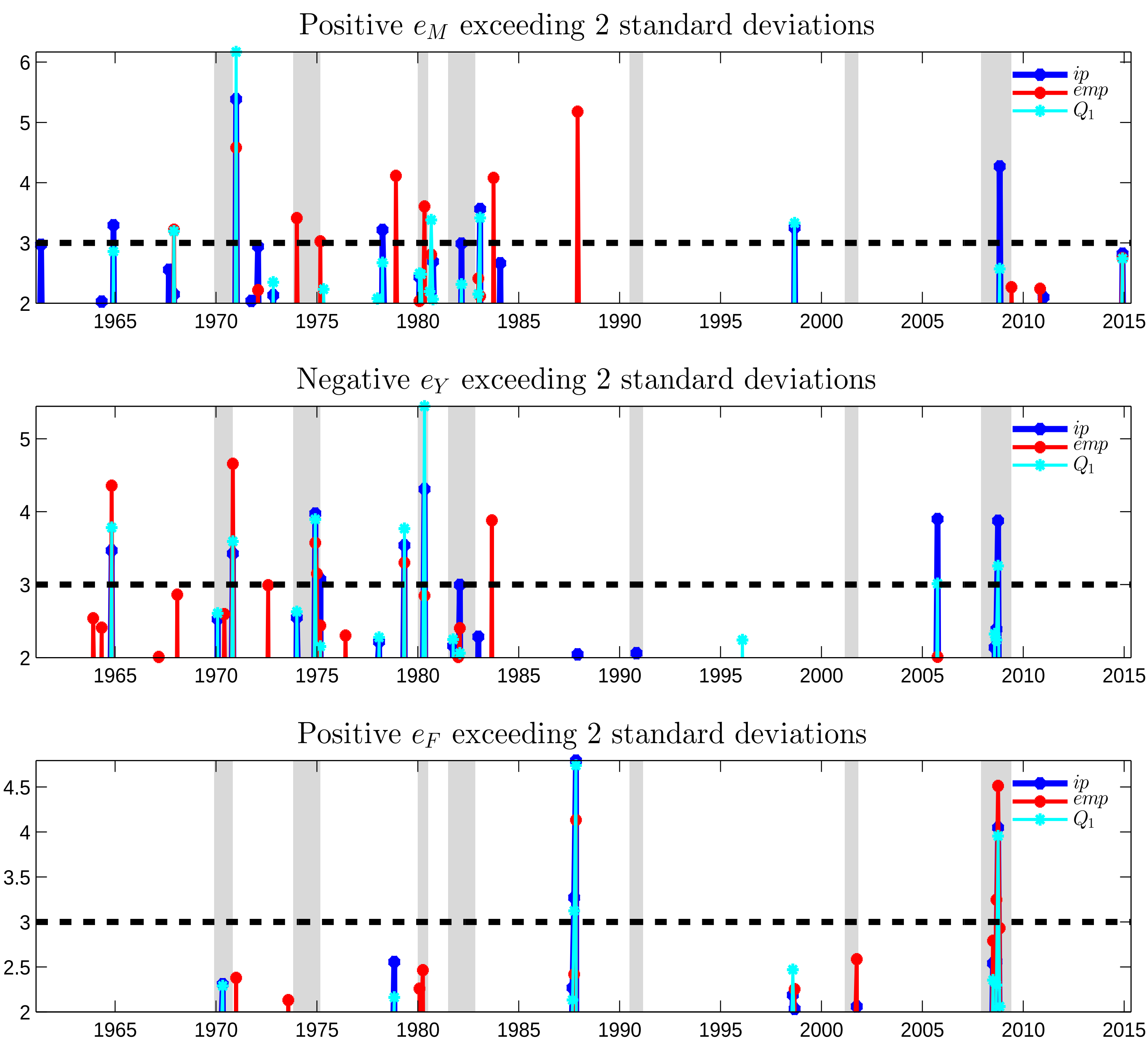
Shaded areas are set of solutions that satisfy $\bar{c} = 0.03$ and $\bar{C} = 0.24$ and event constraints. Dashed line is the max C solution. Responses to positive one stdev shocks are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 4: IRFs of SVAR $(U_M, Y, U_F)'$



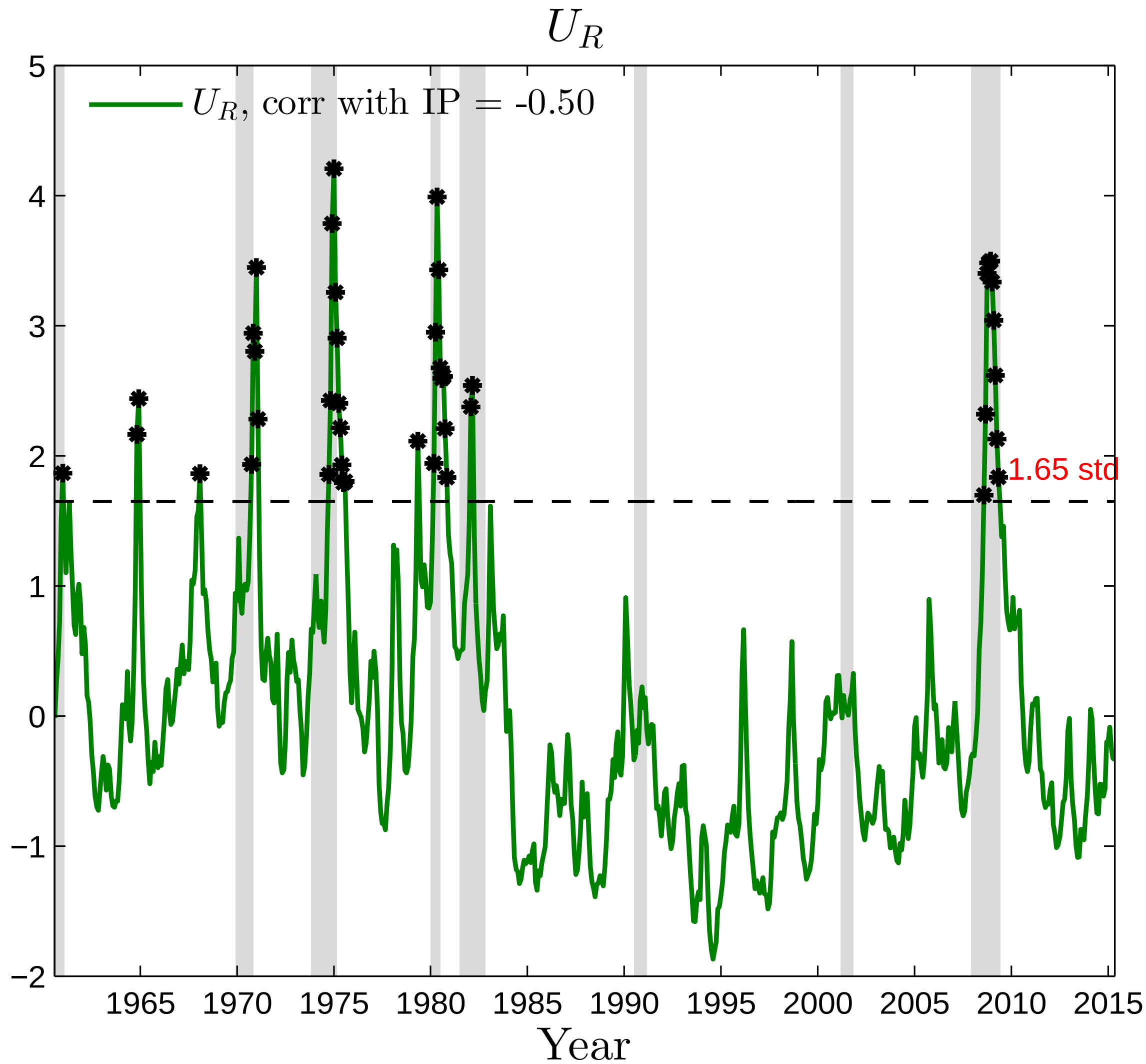
Shaded areas are set of solutions that satisfy $\bar{c} = 0.03$ and $\bar{C} = 0.24$ and event constraints. Dashed line is the max C solution. Responses to positive one stdev shocks are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 5: Large Shock Episodes in SVAR$(U_M, Y, U_F)'$



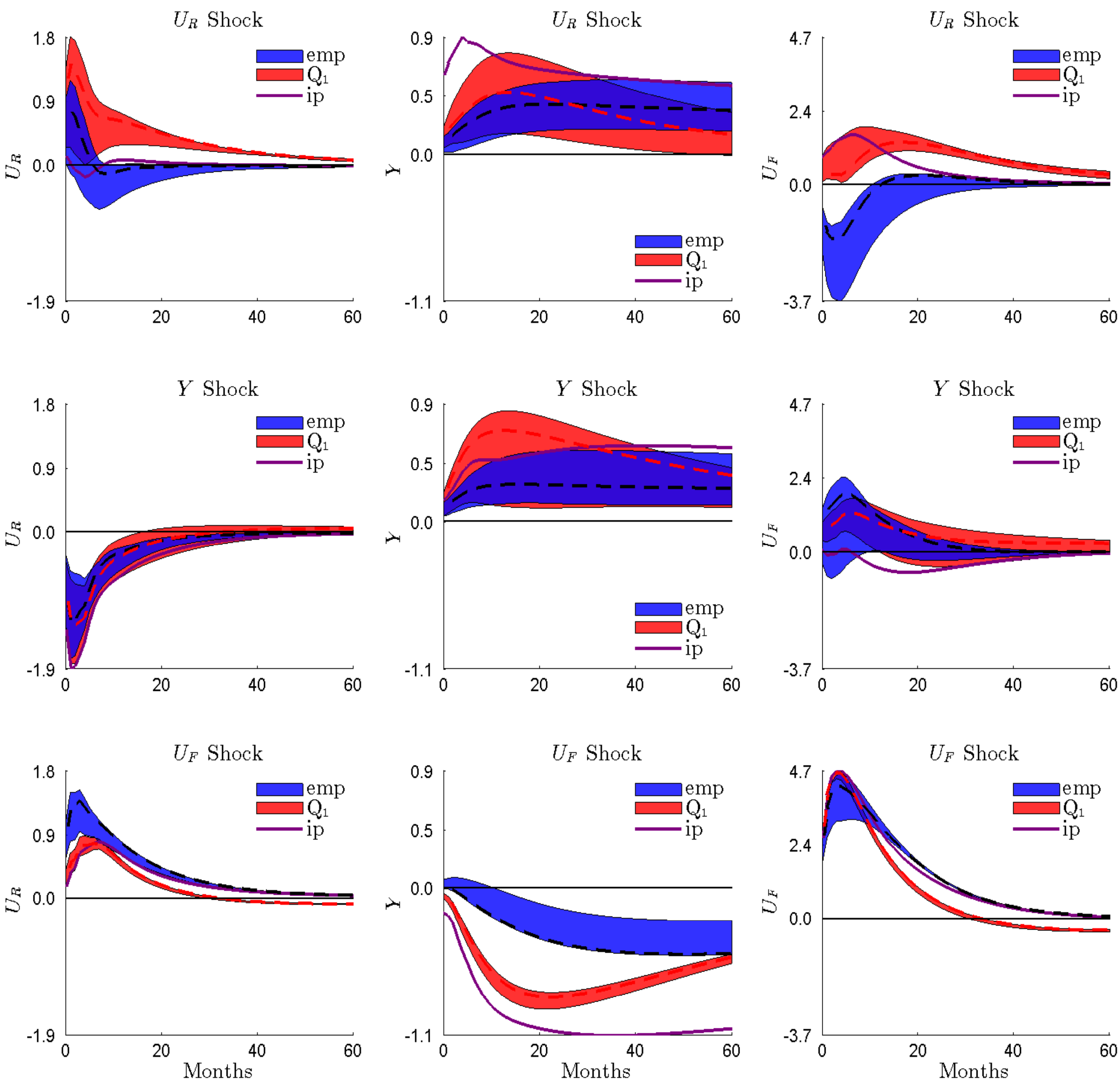
For the max C solution, the figure exhibits shocks that are at least 2 standard deviations above the unconditional mean for e_M and e_F and below for e_Y for three cases where $Y = ip, emp, Q_1$. The shocks $\mathbf{e}_t = \mathbf{B}^{-1}\boldsymbol{\eta}_t$ are reported, where $\boldsymbol{\eta}_t$ is the residual from VAR(6) and $\mathbf{B} = \mathbf{A}^{-1}\mathbf{\Sigma}^{\frac{1}{2}}$. The horizontal line corresponds to 3 standard deviations shocks. The sample spans the period 1960:07 to 2015:04.

Figure 6: Real Uncertainty Over Time



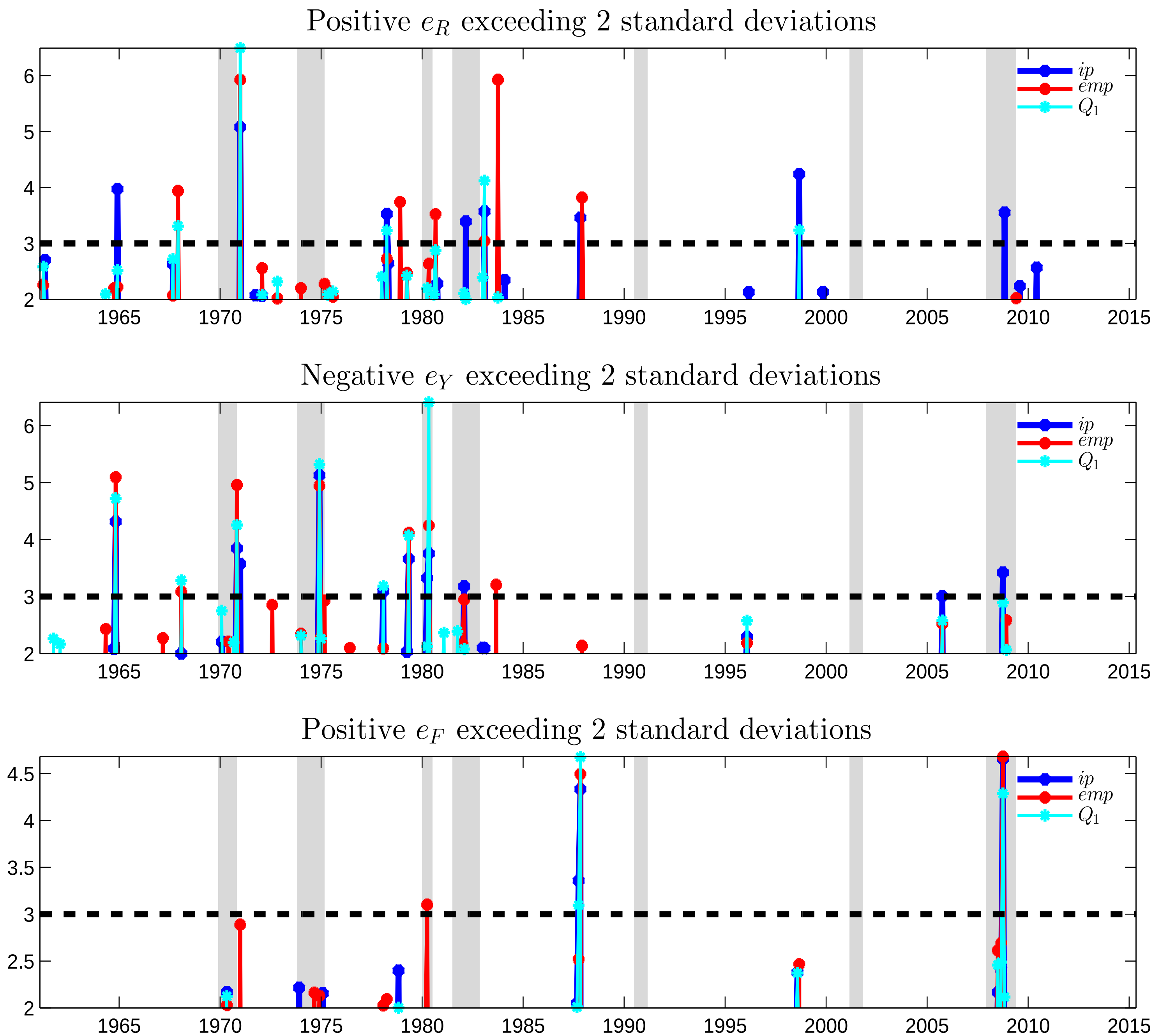
This plot shows time series of U_R, expressed in standardized units. The shaded areas correspond to the NBER recession dates. The horizontal line corresponds to 1.65 standard deviations above the unconditional mean of each series (which has been normalized to zero). Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when U_R is 1.65 standard deviations above its unconditional mean. The data are monthly and span the period 1960:07 to 2015:04.

Figure 7: IRFs of SVAR $(U_R, Y, U_F)'$



Shaded areas are set of solutions that satisfy $\bar{c} = 0.03$ and $\bar{C} = 0.22$ and event constraints. Dashed line is the max C solution. Responses to positive one stdev shocks are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

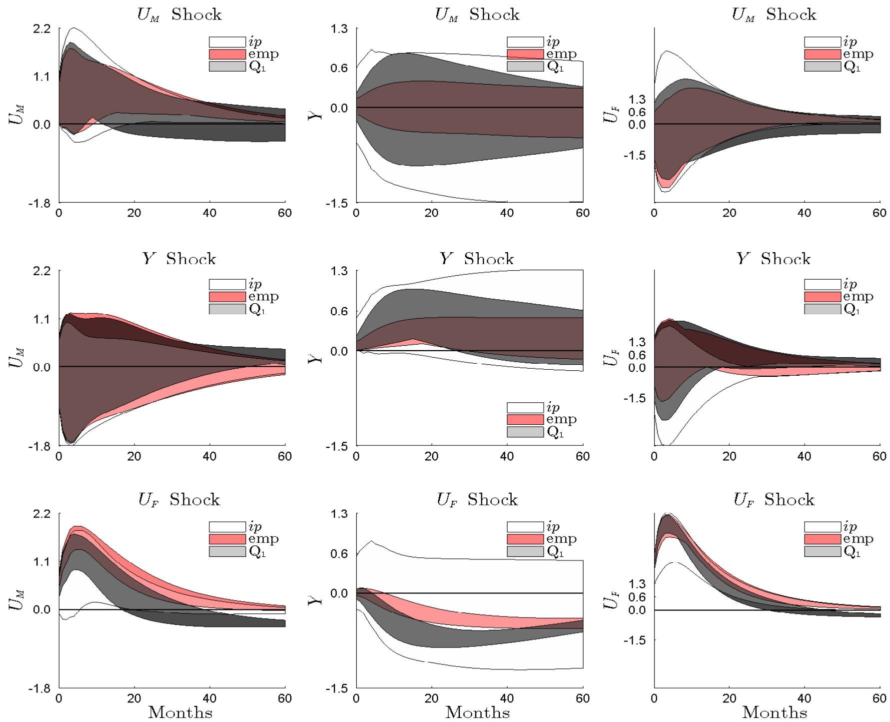
Figure 8: Large Shock Episodes in SVAR$(U_R, Y, U_F)'$



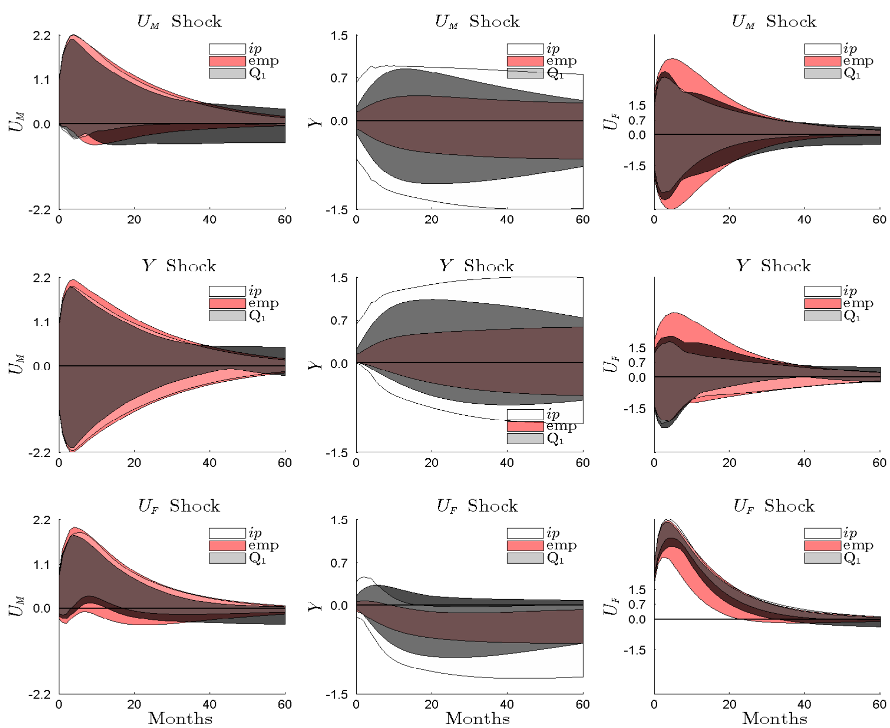
For the max C solution, the figure exhibits shocks that are at least 2 standard deviations above the unconditional mean for e_R and e_F and below for e_Y for three cases where $Y = ip, emp, Q_1$. The shocks $\mathbf{e}_t = \mathbf{B}^{-1}\boldsymbol{\eta}_t$ are reported, where $\boldsymbol{\eta}_t$ is the residual from VAR(6) and $\mathbf{B} = \mathbf{A}^{-1}\boldsymbol{\Sigma}^{\frac{1}{2}}$. The horizontal line corresponds to 3 standard deviations shocks. The sample spans the period 1960:07 to 2015:04.

Figure 9: Set of Solutions with Event or Correlation Constraints Only

Event constraints only

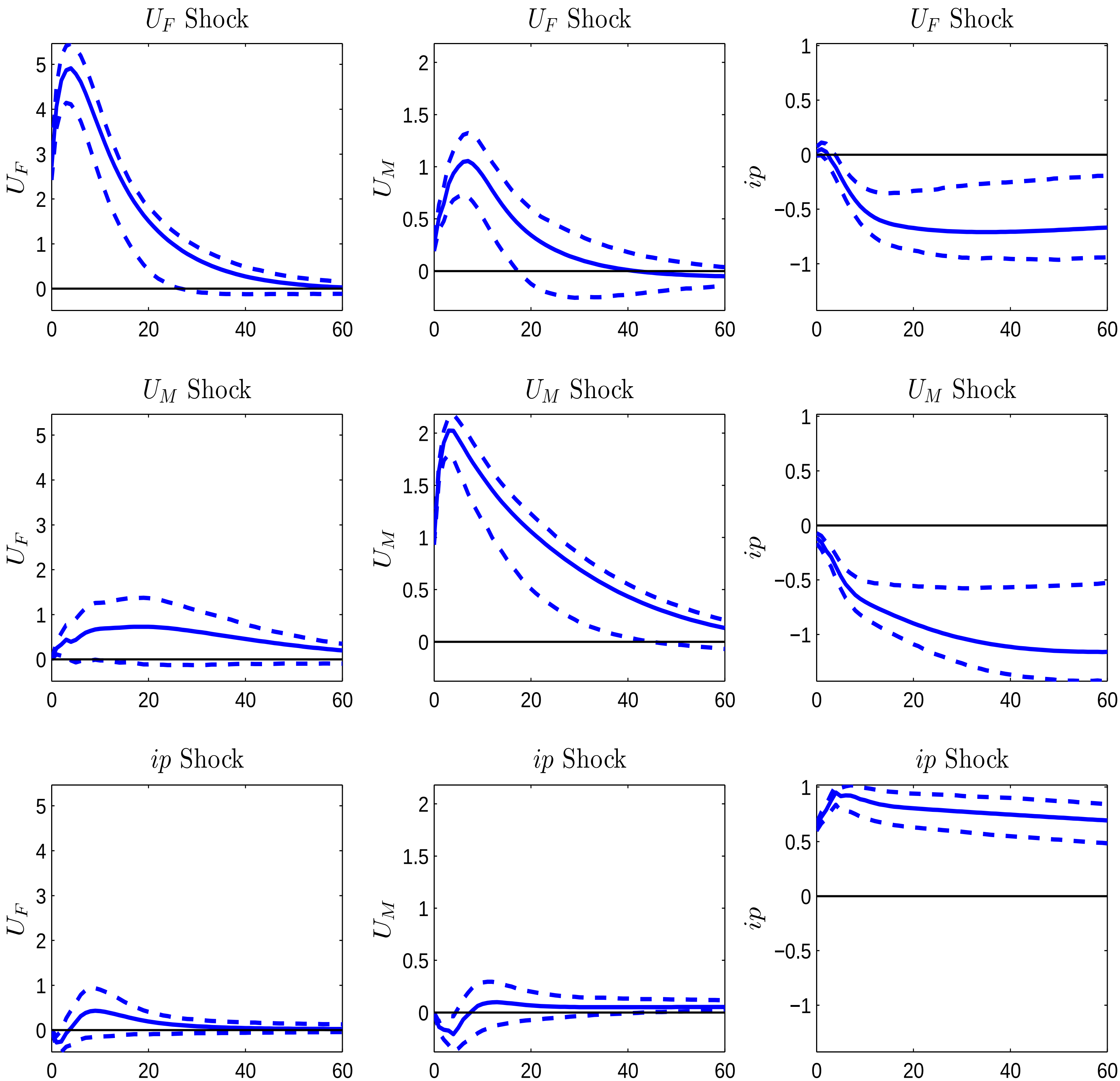


Correlation constraints only



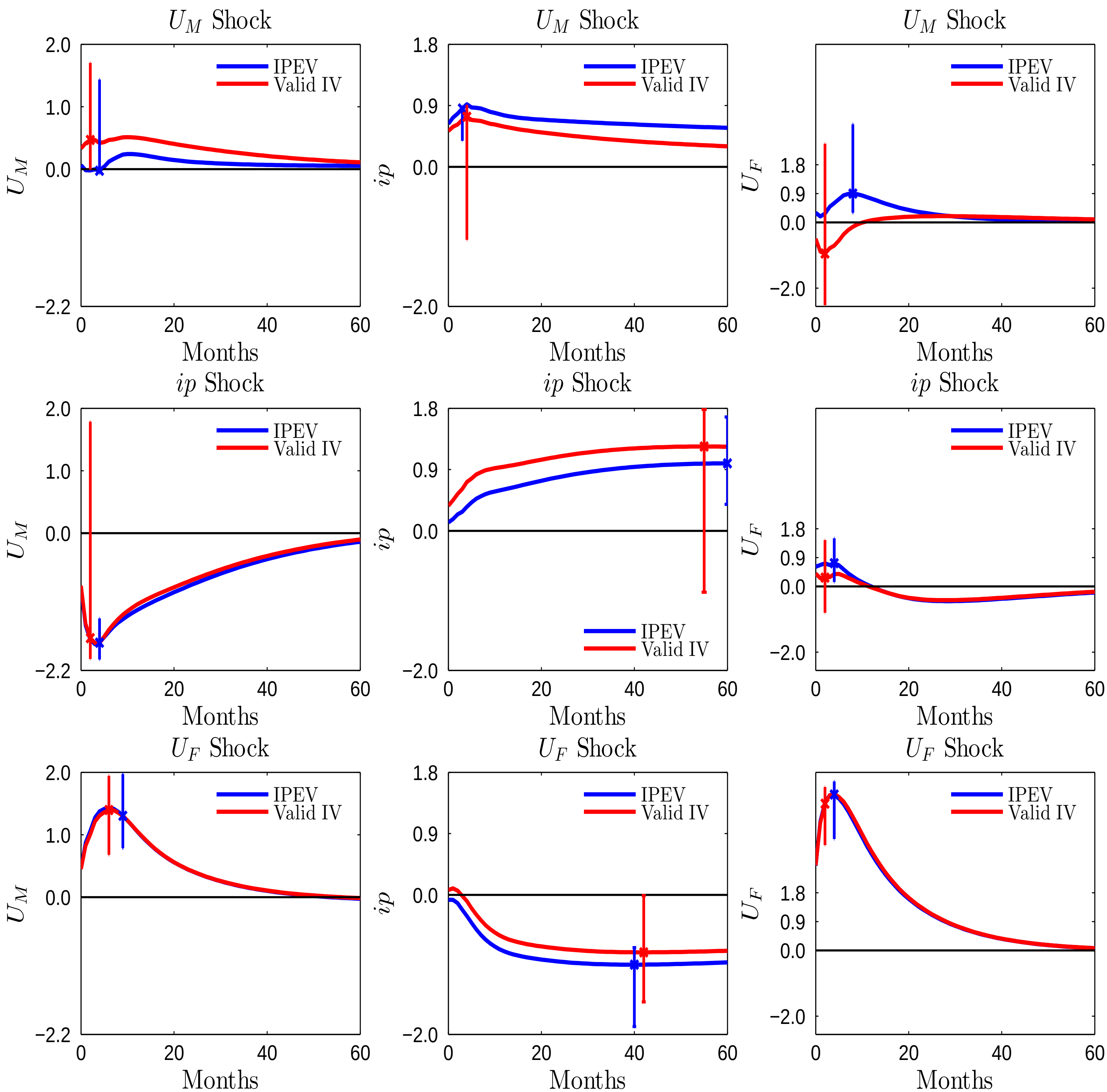
The top panel reports the set of solutions that only satisfy event constraints . The bottom panel reports set of solutions that only satisfy correlation constraints . The sample spans the period 1960:07 to 2015:04.

Figure 10: IRFs using Recursive Identification with Order $(U_F, U_M, ip)'$



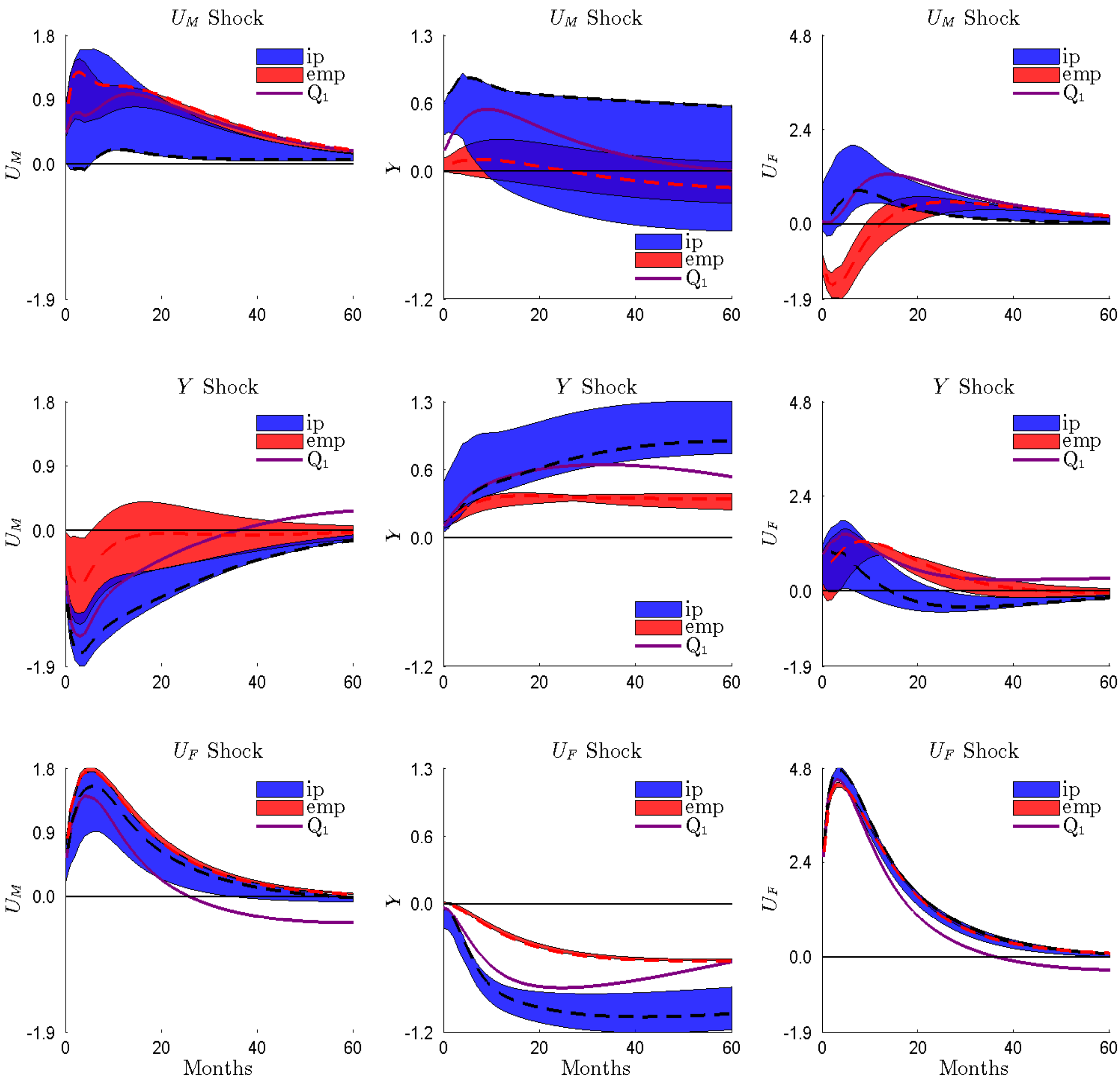
Bootstrapped 90% error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

Figure 11: IRFs of SVAR $(U_M, ip, U_F)'$, ex-post valid IV v.s. IPEV



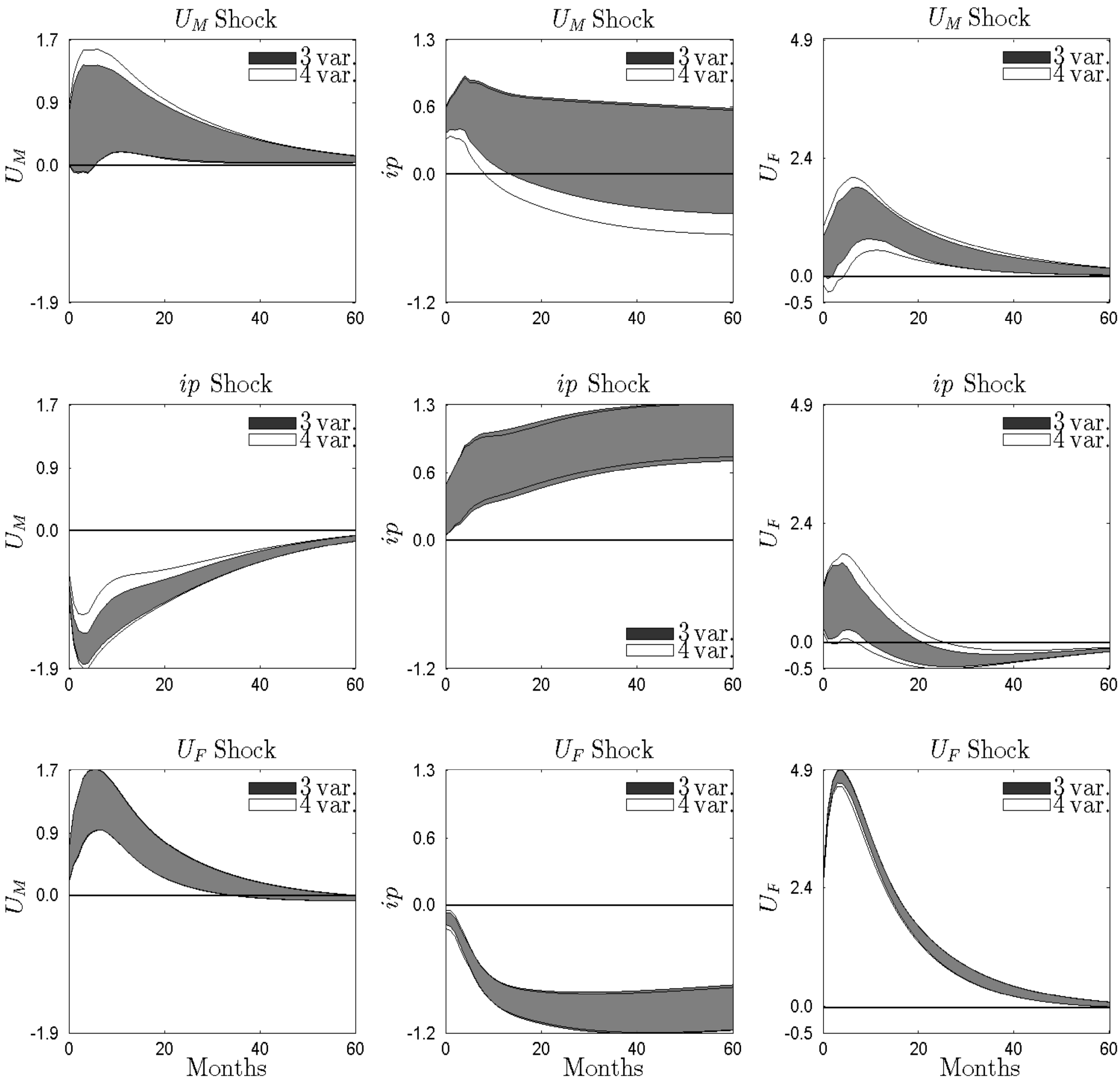
The figure displays impulse responses to one standard deviation shocks. For IPEV, it reports the max C solution. Response units are reported in percentage points. Bootstrapped 90% error bands appear as vertical lines. Ex-post valid IV uses $Z_1 = U_{\text{SPX}}$ and $Z_2 = r_{\text{CRSP}}$. IPEV uses $S_1 = U_{\text{SPX}}$ and $S_2 = r_{\text{CRSP}}$. The sample spans the period 1960:07 to 2015:04

Figure 12: IRFs of SVAR $(U_M, Y, U_F, r_{\mathrm{CRSP}})'$



Shaded areas are set of solutions that satisfy $\bar{c} = 0.03$ and $\bar{C} = 0.24$ and event constraints. Dashed lines are the max C solution. The sample spans the period 1960:07 to 2015:04.

Figure 13: IRFs of SVAR $(U_M, ip, U_F, r_{\text{CRSP}})'$ v.s. $(U_M, ip, U_F)'$



Shaded areas are set of solutions that satisfy $\bar{c} = 0.03$ and $\bar{C} = 0.24$ and event constraints. The sample spans the period 1960:07 to 2015:04.

Table 1: Sample Statistics

Panel A: Correlations between Instruments and Shocks			
$SVAR$	$(U_M, ip, U_F)'$	$(U_M, emp, U_F)'$	$(U_M, Q_1, U_F)'$
$\rho\left(Z_{1t}(\boldsymbol{\beta}), \hat{e}_{Mt}\right)$	$\mathbf{-0.0352}$	$\mathbf{-0.0746}$	$\mathbf{-0.0708}$
	(0.0019)	(0.0040)	(0.0039)
$\rho\left(Z_{1t}(\boldsymbol{\beta}), \hat{e}_{Ft}\right)$	$\mathbf{-0.1845}$	$\mathbf{-0.1784}$	$\mathbf{-0.1745}$
	(0.0101)	(0.0098)	(0.0095)
$\rho\left(Z_{2t}(\boldsymbol{\beta}), \hat{e}_{Ft}\right)$	$\mathbf{-0.1634}$	$\mathbf{-0.1532}$	$\mathbf{-0.1593}$
	(0.0099)	(0.0093)	(0.0096)
$\rho\left(Z_{1t}(\boldsymbol{\beta}), \hat{e}_{Yt}\right)$	0.0000	0.0000	0.0000
$\rho\left(Z_{2t}(\boldsymbol{\beta}), \hat{e}_{Yt}\right)$	0.0000	0.0000	0.0000
$\rho\left(Z_{2t}(\boldsymbol{\beta}), \hat{e}_{Mt}\right)$	0.0000	0.0000	0.0000
Panel B: Estimates of $\boldsymbol{\Sigma}$			
σ_{MM}	**0.0039**	**0.0059**	**0.0060**
	(0.0016)	(0.0012)	(0.0004)
	$[0.001, 0.007]$	$[0.004, 0.008]$	$[0.004, 0.008]$
σ_{YY}	**0.0035**	**0.0013**	**0.0014**
	(0.0010)	(0.0001)	(0.0001)
	$[0.000, 0.005]$	$[0.001, 0.001]$	$[0.001, 0.002]$
σ_{FF}	**0.0265**	**0.0232**	**0.0264**
	(0.0030)	(0.0032)	(0.0029)
	$[0.018, 0.027]$	$[0.016, 0.026]$	$[0.017, 0.027]$

For the max C solution, panel A reports the correlation between the estimated uncertainty shocks and the instruments. Panel B reports estimates of $\boldsymbol{\Sigma}$ that give the standard deviation of each structural shock. Asymptotic standard errors are reported in brackets and bootstrapped 90 percent confidence intervals are reported in parentheses. Bold numbers indicate statistical significance at 10 percent level. The data are monthly and span the period 1960:07 to 2015:04.

Table 2: Variance Decomposition for SVARs in System $(U_M, Y, U_F)'$

	SVAR $(U_M, ip, U_F)'$			SVAR $(U_M, emp, U_F)'$			SVAR $(U_M, Q_1, U_F)'$		
	Fraction variation in U_M			Fraction variation in U_M			Fraction variation in U_M		
s	U_M Shock	ip Shock	U_F Shock	U_M Shock	emp Shock	U_F Shock	U_M Shock	Q_1 Shock	U_F Shock
1	0.107	0.669	0.224	0.293	0.151	0.555	0.310	0.566	0.124
12	0.111	0.512	0.377	0.232	0.072	0.696	0.367	0.427	0.207
∞	0.142	0.534	0.324	0.350	0.048	0.602	0.444	0.350	0.206
$s_{\max}$	0.147	0.669	0.377	0.350	0.151	0.697	0.518	0.566	0.215
	$[0.01, 0.46]$	$[0.45, 0.78]$	$[0.17, 0.67]$	$[0.18, 0.73]$	$[0.06, 0.41]$	$[0.33, 0.79]$	$[0.25, 0.81]$	$[0.28, 0.71]$	$[0.13, 0.58]$
	Fraction variation in ip			Fraction variation in emp			Fraction variation in Q_1		
s	U_M Shock	ip Shock	U_F Shock	U_M Shock	emp Shock	U_F Shock	U_M Shock	Q_1 Shock	U_F Shock
1	0.678	0.317	0.005	0.202	0.797	0.000	0.547	0.415	0.038
12	0.316	0.457	0.227	0.129	0.677	0.194	0.235	0.489	0.276
∞	0.048	0.563	0.389	0.025	0.330	0.645	0.057	0.602	0.341
$s_{\max}$	0.697	0.563	0.394	0.209	0.810	0.645	0.558	0.602	0.374
	$[0.44, 0.92]$	$[0.12, 0.65]$	$[0.28, 0.88]$	$[0.12, 0.52]$	$[0.58, 0.95]$	$[0.29, 0.91]$	$[0.29, 0.73]$	$[0.30, 0.75]$	$[0.25, 0.81]$
	Fraction variation in U_F			Fraction variation in U_F			Fraction variation in U_F		
s	U_M Shock	ip Shock	U_F Shock	U_M Shock	emp Shock	U_F Shock	U_M Shock	Q_1 Shock	U_F Shock
1	0.001	0.028	0.971	0.190	0.051	0.759	0.021	0.022	0.956
12	0.014	0.018	0.968	0.107	0.095	0.798	0.091	0.035	0.874
∞	0.029	0.028	0.943	0.104	0.111	0.785	0.176	0.078	0.746
$s_{\max}$	0.029	0.048	0.978	0.201	0.113	0.799	0.181	0.078	0.959
	$[0.01, 0.30]$	$[0.03, 0.29]$	$[0.74, 0.98]$	$[0.06, 0.38]$	$[0.01, 0.44]$	$[0.63, 0.96]$	$[0.03, 0.38]$	$[0.07, 0.56]$	$[0.69, 0.98]$

For the max C solution, each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$s = s_{max}$" reports the maximum fraction (across all VAR forecast horizons m) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.

Table 3: Variance Decomposition for SVARs in System $(U_R, Y, U_F)'$

	SVAR $(U_R, ip, U_F)'$			SVAR $(U_R, emp, U_F)'$			SVAR $(U_R, Q_1, U_F)'$		
	Fraction variation in U_R			Fraction variation in U_R			Fraction variation in U_R		
s	U_R Shock	ip Shock	U_F Shock	U_R Shock	emp Shock	U_F Shock	U_R Shock	Q_1 Shock	U_F Shock
1	0.003	0.970	0.027	0.176	0.407	0.417	0.615	0.317	0.068
12	0.004	0.787	0.208	0.073	0.321	0.605	0.473	0.306	0.221
∞	0.005	0.743	0.252	0.064	0.288	0.648	0.515	0.262	0.223
$s_{\max}$	0.008	0.978	0.252	0.220	0.411	0.648	0.659	0.360	0.226
	$[0.01, 0.12]$	$[0.80, 1.00]$	$[0.06, 0.60]$	$[0.09, 0.49]$	$[0.25, 0.66]$	$[0.38, 0.75]$	$[0.51, 0.79]$	$[0.20, 0.52]$	$[0.06, 0.49]$
	Fraction variation in ip			Fraction variation in emp			Fraction variation in Q_1		
s	U_R Shock	ip Shock	U_F Shock	U_R Shock	emp Shock	U_F Shock	U_R Shock	Q_1 Shock	U_F Shock
1	0.854	0.066	0.080	0.484	0.515	0.001	0.238	0.701	0.061
12	0.506	0.130	0.364	0.478	0.375	0.147	0.166	0.495	0.340
∞	0.209	0.169	0.622	0.302	0.171	0.527	0.104	0.315	0.581
$s_{\max}$	0.857	0.169	0.622	0.490	0.524	0.527	0.243	0.705	0.581
	$[0.71, 0.98]$	$[0.01, 0.36]$	$[0.38, 0.92]$	$[0.25, 0.72]$	$[0.36, 0.77]$	$[0.16, 0.92]$	$[0.11, 0.43]$	$[0.62, 0.87]$	$[0.24, 0.84]$
	Fraction variation in U_F			Fraction variation in U_F			Fraction variation in U_F		
s	U_R Shock	ip Shock	U_F Shock	U_R Shock	emp Shock	U_F Shock	U_R Shock	Q_1 Shock	U_F Shock
1	0.078	0.001	0.922	0.138	0.113	0.749	0.004	0.056	0.940
12	0.108	0.004	0.889	0.079	0.135	0.786	0.025	0.089	0.886
∞	0.108	0.030	0.862	0.065	0.120	0.815	0.145	0.114	0.741
$s_{\max}$	0.110	0.030	0.928	0.138	0.142	0.815	0.146	0.114	0.944
	$[0.02, 0.37]$	$[0.01, 0.26]$	$[0.74, 0.98]$	$[0.05, 0.40]$	$[0.03, 0.42]$	$[0.57, 0.96]$	$[0.02, 0.39]$	$[0.06, 0.42]$	$[0.84, 0.99]$

For the max C solution, each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted "$s = s_{max}$" reports the maximum fraction (across all VAR forecast horizons m) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples using the procedure described in the Appendix. The data are monthly and span the period 1960:07 to 2015:04.

Table 4: Tests of Validity of Recursive Restriction in System $(U_M, Y, U_F)'$

Ordering:	$(U_M, ip, U_F)'$	$(U_R, ip, U_F)'$
H_0: $B_{RY} = B_{RF} = B_{YF} = 0$	**265.64**	**337.54**
	[147.78]	[83.54]
H_0: $B_{YR} = B_{YF} = B_{RF} = 0$	**383.28**	**457.95**
	[108.25]	[146.74]
H_0: $B_{RY} = B_{RF} = B_{FY} = 0$	**265.49**	**227.58**
	[164.29]	[95.82]
$\chi^2_{5\%}(3)$	7.81	7.81
	$(U_M, emp, U_F)'$	$(U_R, emp, U_F)'$
H_0: $B_{RY} = B_{RF} = B_{YF} = 0$	**316.22**	**9.89**
	[120.73]	[8.48]
H_0: $B_{YR} = B_{YF} = B_{RF} = 0$	**223.98**	**11.03**
	[66.29]	[8.35]
H_0: $B_{RY} = B_{RF} = B_{FY} = 0$	**318.61**	**8.64**
	[121.75]	[9.65]
$\chi^2_{5\%}(3)$	7.81	7.81

For the max C solution, the table reports the Wald test statistic for testing the null hypothesis given in the column. The bold indicates that Wald test rejects the null at 95 percent level according to $\chi^2(3)$ distribution. The SVAR system is solved using GMM and delta method is used for computing the standard error. Estimates of **B** are based on the SVAR identified with external instruments described in the text. The mean of bootstrap Wald statistics is reported in parenthesis. The sample size spans 1960:07 to 2015:04.